SKÁLÁZÁS

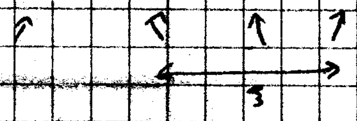
1. óra

1

Phase transitions
↳ 1st , 2nd ..

correlation length : ξ

If we heat up iron



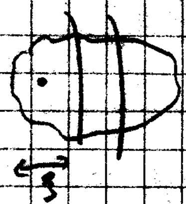
The length in which the directions of spins are the same

$$C(\underline{r},\underline{r}') = \langle \underline{M}(\underline{r})\, \underline{M}(\underline{r}') \rangle - \langle \underline{M}(\underline{r}) \rangle \langle \underline{M}(\underline{r}') \rangle$$

For $T > T_c$ $\langle M \rangle = \emptyset$, but $\langle M\, M \rangle$ is finite

$\sim e^{-|\underline{r}-\underline{r}'|/\xi}$

exponentially fast decay



← macroscopic piece of iron

← chopping off → when we see something change - we have reached ~~the~~ length scale of ξ

1st PT

liquid A gas B

A | B $T = T_c$

they coexist , separate in space

ξ = finite

2nd PT

A $T < T_c$

AB $\underbrace{T = T_c}_{\text{critical phase}}$

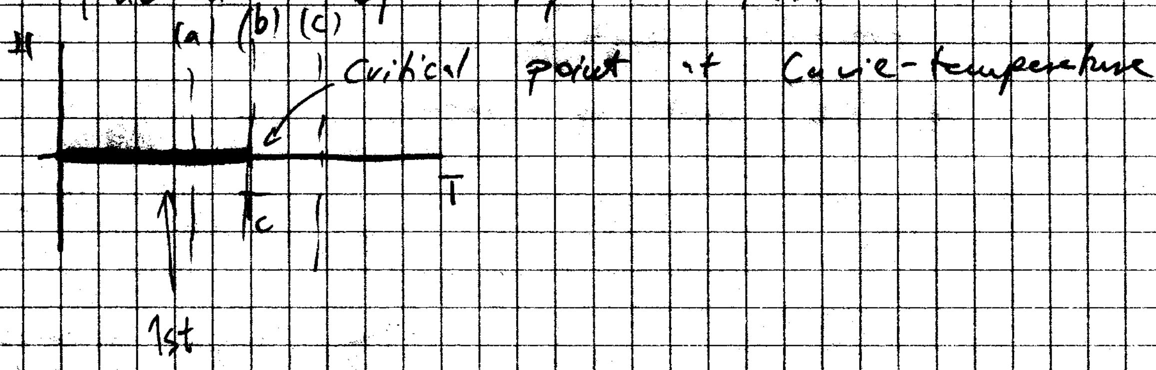
B $T > T_c$

critical phase , $\xi = \infty$

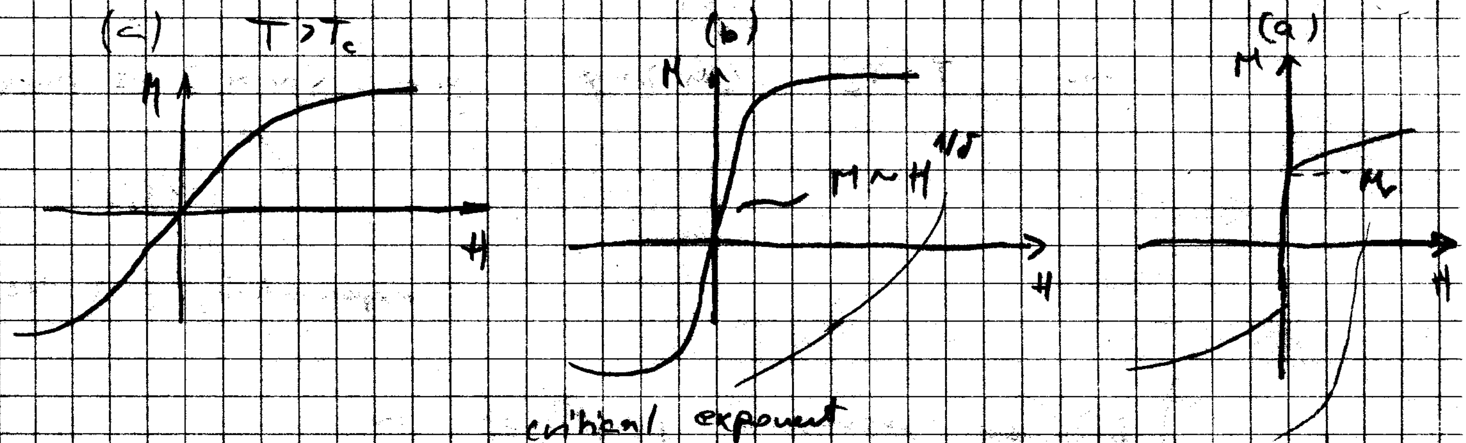
Renormalization group tells us that this $\xi = \infty$ phase is not so difficult

phase diagrams:

in the function of T, H ...



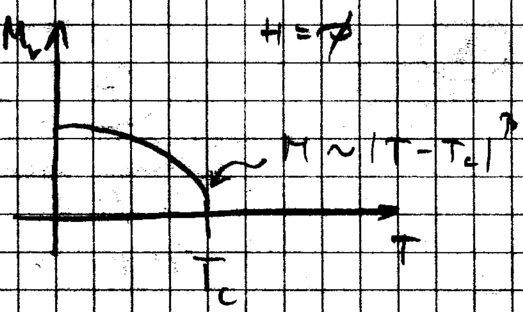
There is an extended region in the plane, that is governed by the $H=0$ $T=T_c$ point



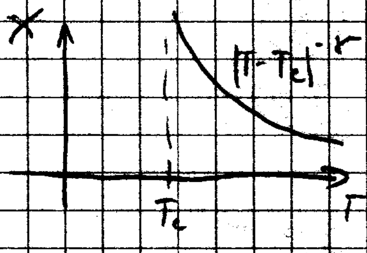
remanent magnetization

hysteresis is always a sign of 1st order PT.

(There is a jump)



susceptibility $\chi = \frac{\partial M}{\partial H}$ $\quad \chi(T, H=0) \leftarrow$ it's the slope of "$M(H)$"



$$\underbrace{\text{It diverges as } T \to T_c}_{\chi(T, H=0) \sim \frac{1}{|T-T_c|^{\gamma}}}$$

Szilárd's | 1. óra (2)

Let's examine the specific heat of the magnet!

$$C_p \sim |T-T_c|^{-\alpha}$$

$$C(\underline{r}-\underline{r}') \sim \frac{1}{|\underline{r}-\underline{r}'|^{d-2+\eta}} \quad \text{at} \quad T=T_c$$

$$\xi \sim |T-T_c|^{-\nu}$$

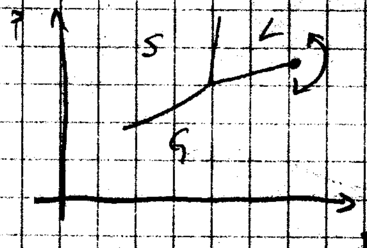
$(\delta, \beta, \gamma, \alpha, \eta, \nu) \leftarrow 6$ critical exponents

One exciting thing is that they are not independent. Typically only two of them are independent.

<u>universality</u>:

- They are independent of microscopic details
- can be the same in very different physical systems

EXAMPLE



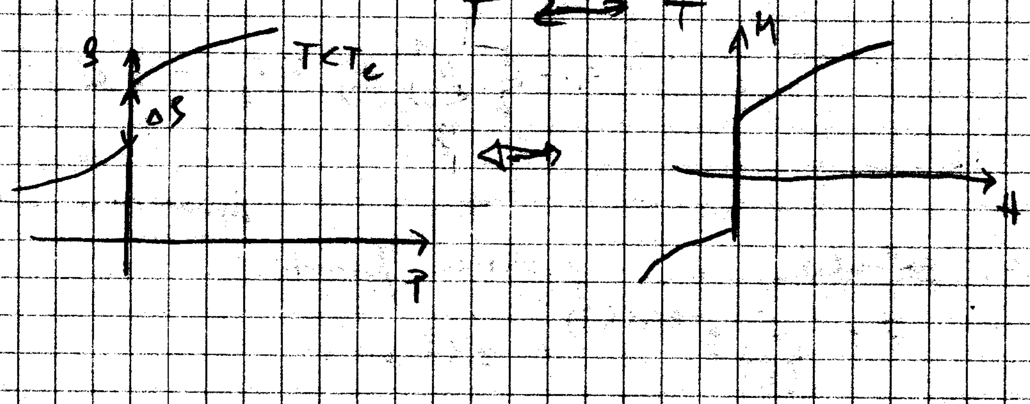
$\longleftrightarrow$ uni-axial magnet

we need to make translation from the magnet example's quantities to the liquid-gas example's

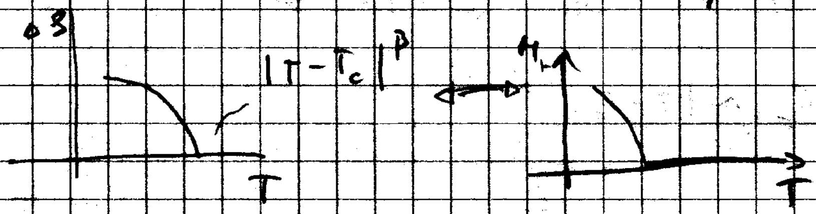
$P \longleftrightarrow H$

$\varsigma \longleftrightarrow M$

$T \longleftrightarrow T$



When we examine the size of the jump:



When we look at the exponents: they are the same

$$\frac{\partial \rho}{\partial p} \sim \chi_c \sim |T-T_c|^{-\gamma} \Longleftrightarrow \chi \sim |T-T_c|^{-\gamma} = \frac{\partial M}{\partial H}$$

compressibility

The critical exponents depend on ~~#~~ 2 things

$\left\{ \begin{array}{l} \text{symmetry} \quad (xy, \ldots) \\ \text{dimensionality} \end{array} \right.$

Why microscopic details don't matter?

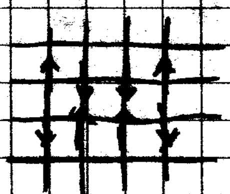
$\xi \to \infty \Rightarrow$ There are no microscopic details

Ising model:

lattice $\underline{r}_i \leftarrow$ index

In each lattice site, there is a spin.

$\sigma_i = \pm 1$



The spins interact with each other

$\downarrow$

$\{\sigma_i\}$ configuration

$$H = -\frac{1}{2} \sum_{ij} J_{ij} \sigma_i \sigma_j - B \sum_i \sigma_i$$

dimension of energy

Let's calculate the partition function:

$$Z = \sum_{\{\sigma_i\}} e^{-\beta H(\{\sigma_i\})} = Z(T,B)$$

Sketch's

$$\beta H \equiv \mathcal{H} = -\frac{1}{2}\sum_{ij} K_{ij}\,\sigma_i\sigma_j - h\sum_j \sigma_j$$

$$K_{ij} = K(r_i - r_j) = \frac{J_{ij}}{T}$$

$k_B = 1$

$\Rightarrow$ $F = -T\log Z$ free energy

<u>Mean field theory:</u>

assume $m = \langle \sigma_i \rangle$

$$(\sigma_i - \underbrace{\langle\sigma_i\rangle}_{m})(\sigma_j - \underbrace{\langle\sigma_j\rangle}_{m}) \approx 0$$

rewrite

$$\sigma_i\sigma_j \to (\sigma_i + \sigma_j)m - m^2$$

Therefore

$$Z = \sum_{\{\sigma_i\}} e^{\frac{1}{2}\sum_{ij} K_{ij}\underbrace{\sigma_i\sigma_j}_{m(\sigma_i+\sigma_j)-m^2} + h\sum_i \sigma_i} \cong \sum_{\{\sigma_i\}} \exp \sum_i \left[\sigma_i(m\mathcal{K}+h) - m^2\frac{1}{2}\mathcal{K}\right]$$

$$\sum_j K_{ij} = \sum_{r_j} K(r_i - r_j) = \sum_{R} K(R) \equiv \mathcal{K} \sim \frac{1}{T}$$

sum over σ_i $\to$ factor $2\cosh(m\mathcal{K}+h)$

$$Z \cong \left(2\cosh(m\mathcal{K}+h)\right)^N e^{-\frac{N}{2}m^2}$$

N: #(lattice sites)

dimensionless free energy density

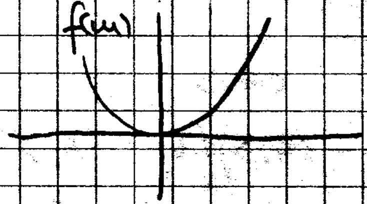
$$\boxed{f \equiv \frac{1}{NT}F = -\frac{1}{N}\ln Z \underset{\text{M.F.}}{\cong} \mathcal{K}\frac{1}{2}m^2 - \ln 2\cosh(m\mathcal{K}+h)}$$

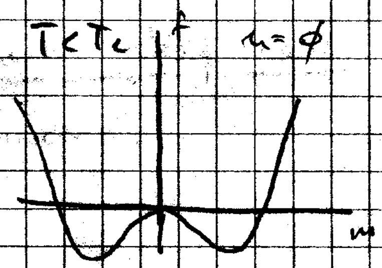
expand in m (assuming that $T \approx T_c$)

$$f = -\cancel{\beta K}/2 + \frac{1}{2}\underbrace{\hat{K}(1-\hat{K})}\, m^2 + \frac{1}{12}\hat{K}^4 m^4 - hm$$

$$\approx \frac{T - T_c^{MF}}{T_c^{MF}} \equiv t \qquad T_c^{MF} = \sum_r J(r)$$

reduced temperature





$m \leftarrow$ it was a parameter so far

$m \longleftrightarrow \frac{\partial f}{\partial m} = \phi$, f is minimal

$\Updownarrow$

$$m = \tanh(\hat{K}m + h)$$

now let's approximate f

$$f \approx \frac{1}{2}tm^2 + \frac{1}{12}m^4 - hm$$

$\Downarrow$

$m(t) \longrightarrow h = \phi : \quad tm + \frac{1}{3}m^3 = \phi$

$\Downarrow$

$m \sim \sqrt{-t}$

$$\beta = \frac{1}{2}, \; \delta = 3, \; \gamma = 1, \; \alpha = \phi, \; \eta = \phi, \; \nu = \frac{1}{2}$$

jump of specific heat

how to get η, ν?

$h \to h_i$

$$\mathcal{H} = -\frac{1}{2}\sum_{ij} J_{ij}\sigma_i\sigma_j - \sum_i h_i\sigma_i$$

$$\langle\sigma_i\rangle = \frac{1}{Z}\sum_{\{\sigma_i\}} \sigma_i \, e^{\frac{1}{2}\sum_{ij}\ldots + \sum_i h_i\sigma_i} = \frac{\partial}{\partial h_i}\ln Z$$

$$\chi_{ij} = \frac{\partial \langle \sigma_i \rangle}{\partial h_j}$$

$\chi_{ij} = \langle \sigma_i \sigma_j \rangle - \langle \sigma_i \rangle \langle \sigma_j \rangle \quad \leftarrow$ This is C_{ij} by definition

FLUCTUATION DISSIPATION THEORY

$$m_i = th\Big(h_i + \underbrace{\sum_j K_{ij} m_j}_{\text{neighbours}}\Big)$$

we assume that $h_i = h + \delta h_i$, where δh_i is a small perturbation (almost homogenic field)

$$m_i = m + \delta m_i$$

$$\cancel{m} + \delta m_i = th\Big(h + \delta h_i + \sum_j K_{ij}(m + \delta m_j)\Big) =$$

$$= \underbrace{\cancel{th(h + \sum K m)}}_{= m} + \frac{1}{ch^2(h + \sum K m)}\Big(\delta h_i + \sum_j K_{ij} \delta m_j\Big) + \dots$$

assume $T > T_c$, $h = \phi$

$$\sum_j (\delta_{ij} - K_{ij}) \delta m_j = \delta h_i$$

if $K_{ij} = K(\underline{r}_i - \underline{r}_j) = K(\underline{r}_{ij})$

$\delta m_i = \delta m_{\underline{r}_i}$

$$\delta h_{\underline{q}} \equiv \frac{1}{\sqrt{N}} \sum_{\underline{r}} e^{-i\underline{q}\underline{r}} \delta h_{\underline{r}} \implies \text{same for } \delta m_{\underline{q}}$$

$$K(\underline{q}) = \sum_{\underline{r}} e^{-i\underline{q}\underline{r}} K(r)$$

$$\delta h_{\underline{r}} = \sum_{\underline{r}'} \big(\delta_{\underline{r}-\underline{r}'} - K(\underline{r}-\underline{r}')\big) \delta m_{\underline{r}'}$$

$$\implies \delta h_{\underline{q}} = (1 - K(\underline{q})) \delta m_{\underline{q}}$$

$$\implies \delta m_{\underline{q}} = \frac{1}{1 - K(\underline{q})} \delta h_{\underline{q}}$$

$$\delta m_{\underline{r}} = \frac{1}{N} \sum_{\underline{q}} \frac{1}{1 - K(\underline{q})} e^{i\underline{q}\underline{r}} \sum_{\underline{r}'} e^{-i\underline{q}\underline{r}'} \delta h_{\underline{r}'}$$

$$\delta m_{\underline{r}} = \sum_{\underline{r}'} \left\{ \frac{1}{N} \sum_{\underline{q}} \frac{1}{1-K(\underline{q})} e^{i\underline{q}(\underline{r}-\underline{r}')} \right\} \delta h_{r'}$$

It is like a response function. This is the definition of susceptibility, but we know, that that is the correlation function

$$K(\underline{q}) = K \left(1 - R_0^2 q^2 + \dots\right)$$

$$T = T_c \Rightarrow K = 1 \Rightarrow C(\underline{q}) \sim \frac{1}{1-K(q)} \sim \frac{1}{q^2} \Rightarrow \eta = 0$$