Egydimenziós mozgás:

$$\boxed{m\ddot{x} = F(x,\dot{x},t)}$$

- $F = F(t)$

$$\dot{x} = v_0 + \frac{1}{m}\int_{t_0}^{t} dt' \, F(t')$$

$$\boxed{x(t) = x_0 + v_0(t-t_0) + \frac{1}{m}\int_{t_0}^{t} dt' \int_{t_0}^{t'} dt'' \, F(t'')}$$

- $F = F(\dot{x})$ (pl. $-\eta\dot{x}$)

$$\dot{v} = \frac{1}{m}F(v) \Rightarrow t - t_0 = \int_{v_0}^{v(t)} dv \frac{m}{F(v)} \equiv Q(v)$$

$$v = Q^{-1}(t-t_0) \Rightarrow x = x_0 + \int_{t_0}^{t} dt' \, Q^{-1}(t-t_0, v_0)$$

$$\Big\{ \text{pl. } t - t_0 = -\frac{m}{\eta}\ln\left(\frac{v}{v_0}\right) \Rightarrow v = v_0 e^{-\frac{\eta}{m}(t-t_0)}$$

$$\Rightarrow x(t) = x_0 + \frac{v_0 m}{\eta}\left(1 - e^{-\frac{\eta}{m}(t-t_0)}\right) \Big\}$$

- $F = F(x)$

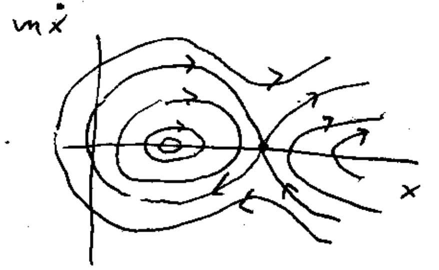
$$V(x) \equiv -\int_{x_0}^{x} dx' \, F(x') \qquad F(x) = -\frac{\partial V}{\partial x}$$

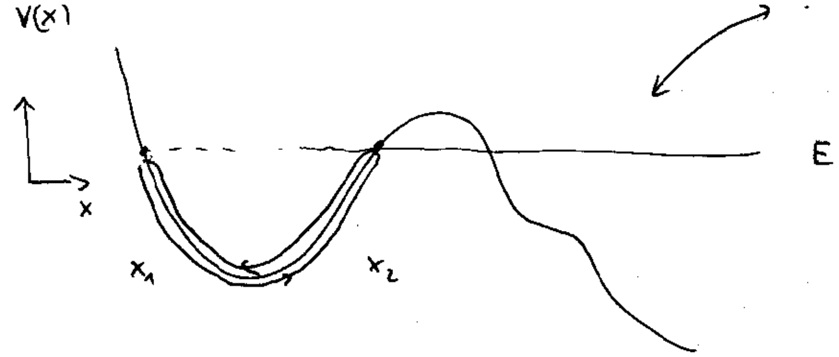
$$\frac{d}{dt}\left(m\frac{\dot{x}^2}{2} + V(x)\right) = m\ddot{x}\dot{x} + \frac{\partial V}{\partial x}\cdot\dot{x} = (m\ddot{x} - F)\dot{x} = 0$$

$$\boxed{E = m\frac{\dot{x}^2}{2} + V(x) = \text{cst}}$$

$$\Rightarrow \dot{x} = \pm\sqrt{\frac{2}{m}}\sqrt{E - V(x)} \Rightarrow \boxed{t - t_0 = \pm\int_{x_0}^{x}\sqrt{\frac{m}{2(E-V(x))}}\,dx}$$

periodikus mozgás:





$x_{1,2}$-ben $\quad \dot{x} = 0 \quad \Rightarrow \quad$ megállási pont $\quad V(x_1) = V(x_2) = E$

$$\Rightarrow \quad \boxed{T(E) = \int_{x_1}^{x_2} \sqrt{\frac{2m}{E - V(x)}}\, dx}$$

integrál divergál, ha $\quad V'(x_2) = \emptyset \; [\, V'(x_1) = 0 \,]$

<u>Harmonikus</u> <u>oszcillátor</u>; <u>mozgás</u> a <u>minimum</u> <u>közelében</u>

$$V(x) = V_0 + \frac{m\omega_0^2}{2} x^2 + \dots \quad \Leftrightarrow \quad \ddot{x} + \omega_0^2 x = 0$$

csillapított + időben gerjesztett:

$$\boxed{\ddot{x} + 2\gamma \dot{x} + \omega_0^2 x = \frac{1}{m} F(t) \equiv f(t)}$$

hom. egy: $\quad x \sim e^{-\lambda t} \quad \left(x = \mathrm{Re}\{\alpha e^{-\lambda t}\} \right)$

$$\lambda^2 - 2\gamma \cdot \lambda + \omega_0^2 = 0 \quad \Rightarrow \quad \lambda_{1,2} = +\gamma \pm \sqrt{\gamma^2 - \omega_0^2}$$

$$\omega_0^2 > \gamma^2 \Rightarrow \pm i \underbrace{\sqrt{\omega_0^2 - \gamma^2}}_{\Omega}$$

$$\Rightarrow x(t) = \alpha_1 e^{-\lambda_1 t} + \alpha_2 e^{-\lambda_2 t}$$

inhom. egy: partik. m.o.

$$\frac{d}{dt} \begin{pmatrix} \dot{x} \\ x \end{pmatrix} = \begin{pmatrix} -2\gamma & -\omega_0^2 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} \dot{x} \\ x \end{pmatrix} + \begin{pmatrix} f(t) \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} \dot{x} \\ x \end{pmatrix} = \alpha_1(t) \begin{pmatrix} \dot{x}_1 \\ x_1 \end{pmatrix} + \alpha_2(t) \begin{pmatrix} \dot{x}_2 \\ x_2 \end{pmatrix} \qquad \text{ansatz}$$

$$\Rightarrow \quad \dot{\alpha}_1 \begin{pmatrix} \dot{x}_1 \\ x_1 \end{pmatrix} + \dot{\alpha}_2 \begin{pmatrix} \dot{x}_2 \\ x_2 \end{pmatrix} = \begin{pmatrix} f(t) \\ 0 \end{pmatrix} \quad \Rightarrow \quad \begin{pmatrix} \dot{x}_1 & \dot{x}_2 \\ x_1 & x_2 \end{pmatrix} \begin{pmatrix} \dot{\alpha}_1 \\ \dot{\alpha}_2 \end{pmatrix} = \begin{pmatrix} f(t) \\ 0 \end{pmatrix}$$

$$\Rightarrow \quad \frac{d}{dt}\begin{pmatrix}\alpha_1\\ \alpha_2\end{pmatrix} = \frac{1}{\dot{x}_1 x_2 - \dot{x}_2 x_1}\begin{pmatrix} x_2 & -\dot{x}_2 \\ -x_1 & \dot{x}_1\end{pmatrix}\begin{pmatrix} f \\ 0\end{pmatrix}$$

$$\dot{\alpha}_1 = f\,\frac{x_2}{\dot{x}_1 x_2 - \dot{x}_2 x_1} = f(t)\,\frac{e^{\lambda_1 t}}{\lambda_2 - \lambda_1}$$

$$\boxed{x_{part}(t) = \mathrm{Re}\left\{\int_{t_0}^{t} dt'\,\frac{e^{-\lambda_1 (t-t')}}{\lambda_2 - \lambda_1}\,f(t') + \text{“}1 \leftrightarrow 2\text{”}\right\}}$$

kauzalitás!

Harmonikus gerjesztésre adott válasz:

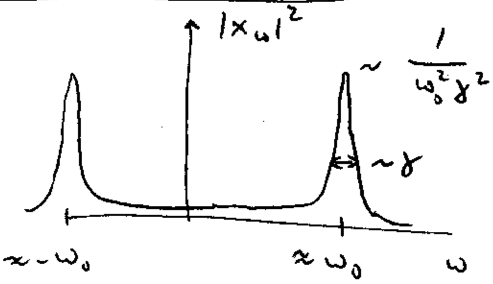
$$f(t) = \mathrm{Re}\{f_\omega e^{-i\omega t}\}$$

aszimptotikusan partikuláris m.o.-hoz konv.

$$x(t) = \mathrm{Re}\{x_\omega e^{-i\omega t}\}$$

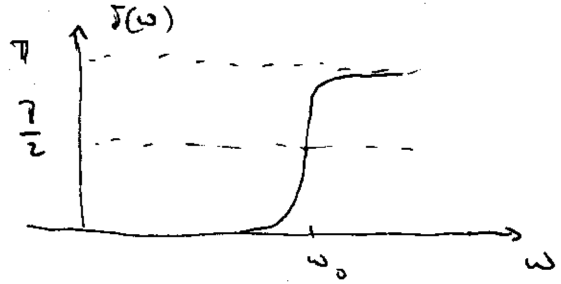
$$x_\omega(-\omega^2 - 2\omega i\gamma + \omega_0^2) = f_\omega \quad \Rightarrow \quad \boxed{x_\omega = \frac{f_\omega}{\omega_0^2 - \omega^2 - 2\omega i\gamma}}$$

amplitúdó: $$|x_\omega|^2 = \frac{|f_\omega|^2}{(\omega_0^2-\omega^2)^2 + 4\omega^2\gamma^2}$$



fázis: $$x_\omega = f_\omega \frac{1}{[(\omega_0^2-\omega^2)^2 + 4\omega^2\gamma^2]^{1/2}}\,e^{i\delta} \quad \Rightarrow \quad \mathrm{tg}\,\delta = \frac{2\omega\gamma}{\omega_0^2 - \omega^2}$$

$$\mathrm{ctg}\,\delta = \frac{\omega_0^2 - \omega^2}{2\omega\gamma}$$

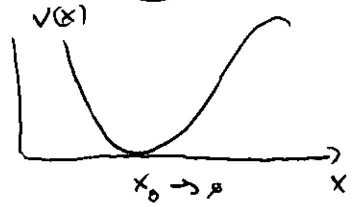


$$f(t) = \underbrace{|f_\omega|}_{\text{valós}}\cos(\omega t) \;\rightarrow\; x(t) = |x_\omega|\cos(\omega t - \delta)$$

fáziskésés! „mérhető”

H.F. $\Rightarrow$ oszcillátor által disszipált energia?

- **Anharmonikus oszcillátor:**



$$V(x) \approx V_0 + \frac{m\omega_0^2}{2} x^2 + \frac{m}{3}\epsilon x^3 + \dots$$

$$\Rightarrow \quad \ddot{x} + \omega_0^2 x + \epsilon x^2 = \emptyset$$

$$x = a\cos(\omega_0 t) + \delta x(t)$$

$$\Rightarrow \quad \delta\ddot{x} + \omega_0^2 \delta x = -\epsilon a^2 \cos^2(\omega_0 t) = -\frac{\epsilon a^2}{2}\left(1 + \cos(2\omega_0 t)\right)$$

$$\Rightarrow \quad \delta x = -\frac{\epsilon a^2}{2\omega_0^2} + y \ : \quad \ddot{y} + \omega_0^2 y = -\frac{\epsilon a^2}{2} e^{-2i\omega_0 t}$$

$$-3\omega_0^2 y = -\frac{\epsilon a^2}{2} e^{-i2\omega_0 t}$$

$$\Rightarrow \quad \boxed{x(t) = a\cos(\omega_0 t) - \frac{\epsilon a^2}{2\omega_0^2} + \frac{\epsilon a^2}{6\omega_0^2}\cos(2\omega_0 t) + \dots}$$

$$x^{(2)} \sim a^2$$

általában:

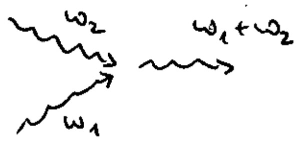
$$x = x^{(1)} + x^{(2)} + x^{(3)} + \dots$$

$$V = \frac{m\omega_0^2}{2} x^2 + \frac{m\epsilon}{3} x^3 + \frac{m\mu}{4} x^4 + \dots$$

$$\omega = \omega_0 + \underbrace{\omega^{(1)}}_{=\,\emptyset} + \omega^{(2)} + \dots$$

$x^{(k)} \sim a^k$, tartalmaz $\cos(k\omega t)$ tagot

$\Rightarrow$ nem lineáris optika!



- **Parametrikus rezonancia:**

$$\frac{d}{dt}(m\dot{x}) + m\omega_0^2 x = 0 \qquad \omega_0(t) \quad m(t)$$

$$\underset{m(0)}{m_0}\frac{d}{d\tau} \equiv m(t)\frac{d}{dt} \quad \Rightarrow \quad \frac{d^2x}{d\tau^2} + \underbrace{\left(\frac{m\omega_0}{m_0}\right)^2}_{\Omega^2(\tau)} x = 0$$

$$\boxed{\frac{d^2x}{d\tau^2} + \Omega^2(\tau) x = 0}$$

t.f.h.: $\Omega(\tau)$ periodikusan változik $\Omega(\tau+T) = \Omega(\tau)$

$\Rightarrow$ $x(\tau)$ megoldás (complex), akkor $x(\tau+T)$ is

két lin. ftl. m.o. létezik: $x_1(\tau)$, $x_2(\tau)$

$$x(\tau) = \alpha_1 x_1(\tau) + \alpha_2 x_2(\tau)$$

$$x_1(\tau+T) = \mu_1 x_1(\tau) \quad ; \quad x_2(\tau+T) = \mu_2 x_2(\tau)$$

$$x_2 \cdot \quad \ddot{x}_1 + \Omega^2 x_1 = 0 \quad \Rightarrow \quad \frac{d}{d\tau}\underbrace{\left(\dot{x}_1 x_2 - \dot{x}_2 x_1\right)}_{W(\tau)} = -\Omega^2(x_1 x_2 - x_2 x_1) = \emptyset$$

$$(\dot{x}_1 x_2 - \dot{x}_2 x_1)(\tau+T) = \mu_1\mu_2 W(\tau) \quad \Rightarrow \quad \boxed{\mu_1\mu_2 = 1}$$

x_1 megoldás $\Rightarrow$ x_1^* is $\Rightarrow$ $\mu_1^* = \mu_1$ vagy $\mu_1^* = \mu_2$

(a) μ_1, μ_2 valós $\mu_1 = \frac{1}{\mu_2}$ $\Rightarrow$ egyik megoldás felrobban!

parametrikus rez.

(b) μ_1 komplex $\Rightarrow$ $\mu_2 = \mu_1^*$ és $|\mu_1|^2 = 1$ ~ stabil megoldások

egyszerű eset:

$$\Omega^2(t) = \omega_0^2\left(1 + \rho\cos(\omega t)\right) \qquad \omega = \frac{2\pi}{T}$$

Mathieu-egyenlet