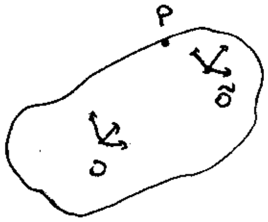
Gyorsuló koordinátarendszerek



$\mathcal{R} = \{o, \{\underline{e}_i\}\}$ inerciarendszer $(o(t), \underline{e}_i(t))$

P tömegpont $\quad \underline{r}(t) = \sum_i x^i(t)\underline{e}_i(t) = \{x^i(t)\}$

$$m\ddot{\underline{r}} = \underline{F}$$

kérdés : $\tilde{\mathcal{R}} = \{\tilde{o}(t), \tilde{\underline{e}}_i(t)\} \rightarrow \overrightarrow{\tilde{o}P} = \tilde{\underline{r}} = \sum_i \tilde{x}^i(t)\tilde{\underline{e}}_i(t)$

milyen mozgásegyenletet elégít ki?

$\overrightarrow{o\tilde{o}} = \underline{R}(t) = \sum_i R^i(t)\underline{e}_i(t)$; $\quad \overrightarrow{oP} = \overrightarrow{o\tilde{o}} + \overrightarrow{\tilde{o}P}$ mindkét von. rendszerben.

általában

$\Rightarrow \overrightarrow{\tilde{o}P} = \sum_i (x^i - R^i)\underline{e}_i$ mi rendszerünkben

a mi rendszerünkben : $\boxed{\tilde{\underline{e}}_i = \sum_k B_{ki}\underline{e}_k} \Rightarrow \boxed{\tilde{A}^i B_{ki} = A^k}$
$\underline{A} = \underline{\underline{B}}\,\tilde{\underline{A}}$

ortogonális mátrix

két pont távolsága u. az $\Leftrightarrow \underline{\underline{B}}\,\underline{\underline{B}}^T = 1$

$\Leftrightarrow \forall\, \tilde{\underline{x}}$-re $\quad \tilde{\underline{x}}^T\underline{\underline{B}}^T\underline{\underline{B}}\,\tilde{\underline{x}} = \tilde{\underline{x}}^2$

$$\boxed{\underline{A}_{PQ} = \underline{\underline{B}}\,\tilde{\underline{A}}_{PQ}}$$

$$\Rightarrow \boxed{\tilde{\underline{x}} = B^T(\underline{x} - \underline{R})}$$

sebesség = ?

$$\dot{\tilde{\underline{x}}} = \dot{B}^T(\underline{x} - \underline{R}) + B^T(\dot{\underline{x}} - \dot{\underline{R}}) = B^T\{\overbrace{(B\dot{B}^T)}^{\Omega}(\underline{x} - \underline{R}) + \dot{\underline{x}} - \dot{\underline{R}}\}$$

de $\quad B^TB = 1 \Rightarrow \dot{B}^TB = -B^T\dot{B} \Rightarrow \dot{B}^T = -B^T\underbrace{\dot{B}B^T}$

$$\Omega^T = (B\dot{B}^T)^T = \dot{B}B^T = -B\dot{B}^T = -\Omega \leftarrow \text{antiszimmetr.}$$

$\boxed{\Omega = B\dot{B}^T}\ \boxed{\Omega_{ij} = -\epsilon_{ikj}\,\omega^k = -(\underline{\omega}\times)_{ij}}$

$$\Rightarrow \boxed{\dot{\tilde{\underline{x}}} = B^T\Big(\underbrace{\dot{\underline{x}} - \dot{\underline{R}} - \underline{\omega}\times(\underline{x} - \underline{R})}_{o \text{ vektortere}}\Big)}$$

$\tilde{o}$ vektortere

általában $\quad \dot{\tilde{\underline{A}}} = B^T(\dot{\underline{A}} - \underline{\omega}\times\underline{A})$

$\Rightarrow \dfrac{\tilde{d}}{dt}\underline{\underline{A}} = \dfrac{d\underline{A}}{dt} - \underline{\underline{\omega}}\times\underline{\underline{A}}$

(pongyola felírás : $B^T \rightarrow 1$, ...

áll. $\quad B^T\Omega = (B^T\Omega B)B^T = -(\tilde{\underline{\omega}}\times)B^T$

$\tilde{\underline{\omega}} = B^T\underline{\omega}$

bit.: $(B^T \Omega B)_{ab} = -B_{ia}\,\epsilon_{ijk}\,\omega^j B_{kb} = \{\epsilon_{ijk} B_{ia} B_{kb} \underbrace{B_{jc}\} B_{ec}}_{(BB^T)_{je}}\omega^e$

$$= \epsilon_{abc}\underbrace{(B^T\underline{\omega})^c}_{\tilde{\underline{\omega}}^c} = (-\tilde{\underline{\omega}}\times)_{ab} \quad ✓$$

követk.:

$$B^T \underset{BB^T}{\Omega}(\underline{x}-\underline{R}) = -(\tilde{\underline{\omega}}\times)(B^T(\underline{x}-\underline{R})) = -\tilde{\underline{\omega}}\times\tilde{\underline{x}} \quad !$$

$$\boxed{\dot{\tilde{\underline{x}}} = -\tilde{\underline{\omega}}\times\tilde{\underline{x}} + B^T(\dot{\underline{x}}-\dot{\underline{R}})}$$

gyorsulás

$$\ddot{\tilde{\underline{x}}} = -\dot{\tilde{\underline{\omega}}}\times\tilde{\underline{x}} - \tilde{\underline{\omega}}\times\dot{\tilde{\underline{x}}} + \underbrace{\dot{B}^T B\, B^T(\dot{\underline{x}}-\dot{\underline{R}})}_{-\tilde{\underline{\omega}}\times(\dot{\tilde{\underline{x}}}+\tilde{\underline{\omega}}\times\tilde{\underline{x}})} + \overbrace{B^T(\ddot{\underline{x}}-\ddot{\underline{R}})}^{B^T\underline{F}/m}$$

$$\boxed{\ddot{\tilde{\underline{x}}} = \frac{1}{m}\tilde{\underline{F}} - \tilde{\ddot{\underline{R}}} + 2\dot{\tilde{\underline{x}}}\times\tilde{\underline{\omega}} - \dot{\tilde{\underline{\omega}}}\times\tilde{\underline{x}} + (\tilde{\underline{x}}\cdot\tilde{\omega}^2 - (\tilde{\underline{\omega}}\cdot\tilde{\underline{x}})\tilde{\underline{\omega}})}$$

$B^T\underline{F}$ külső erő

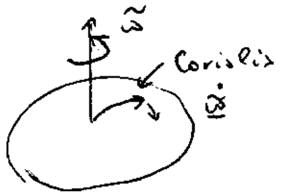
$= B^T\ddot{\underline{R}}$

Coriolis erő/m

centrifugális erő/m

tehetetlenségi erők

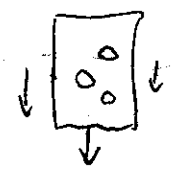
mozgó bcsz...



gravitáció különleges $\quad \tilde{\underline{F}} \propto m \quad \forall$ testre

$\Rightarrow$ szabad esés $\Rightarrow \quad \ddot{\tilde{\underline{R}}} - \frac{1}{m}\tilde{\underline{F}} = 0$

zuhanó lift



$\Rightarrow$ olyan, mintha nem is lenne a inerciarendszer

$\Rightarrow$ ált. rel. elm. !

- Foucault-inga

H.F.: tegyük fel, h. egy űrhajón élünk, nem látunk ki..., Föld körül keringünk $\rightarrow$ fel tudjuk-e állítani a tömegvonzás törvényét??

B verzió: Tilvas levezetése

azonosítsuk a két vektorteret...

$$\vec{r} = \tilde{x}^i \vec{\tilde{e}}_i = (x^i - R^i)\vec{e}_i$$

a mi koordinátarend szerinkben

$$\frac{d\vec{r}}{dt} \underset{\substack{\uparrow \\ \text{definíció szerint}}}{=} \left(\frac{dx^i}{dt} - \frac{dR^i}{dt}\right)\vec{e}_i = \frac{d\tilde{x}^i}{dt}\vec{\tilde{e}}_i + \tilde{x}^i \frac{d\vec{\tilde{e}}_i}{dt}$$

$\vec{\tilde{e}}_i$ viszont forog

áll.: $\exists\, \vec{\omega}$! $\dfrac{d\vec{\tilde{e}}_i}{dt} = \vec{\omega} \times \vec{\tilde{e}}_i$

$$\Rightarrow \frac{d\vec{r}}{dt} = \underbrace{\frac{d\tilde{x}^i}{dt}\vec{\tilde{e}}_i}_{\frac{\tilde{d}\vec{r}}{\tilde{d}t}\ \text{def. szerint}} + \vec{\omega}\times\vec{r} \quad \Rightarrow \quad \boxed{\frac{\tilde{d}\vec{r}}{\tilde{d}t} = \frac{d\vec{r}}{dt} - \vec{\omega}\times\vec{r}}$$

$$\vec{\omega}\times\vec{\tilde{e}}_i = \tilde{\omega}^k\, \vec{\tilde{e}}_k \times \vec{\tilde{e}}_i = \tilde{\omega}^k \epsilon_{kil}\, \vec{\tilde{e}}_l$$

$$\Rightarrow \tilde{x}^i \frac{d\vec{\tilde{e}}_i}{dt} = \tilde{x}^i \tilde{\omega}^k \epsilon_{kil}\, \vec{\tilde{e}}_l = (\underline{\tilde{\omega}} \times \underline{\tilde{x}})^l\, \vec{\tilde{e}}_l$$

$$(\dot{x}^i - \dot{R}^i)\vec{e}_i = \left(\dot{\tilde{x}}^i + (\underline{\tilde{\omega}}\times\underline{\tilde{x}})^i\right)\vec{\tilde{e}}_i$$

máshogy:

$$\vec{\omega}\times\vec{r} = \omega^i (x^j - R^j)\, \underbrace{\vec{e}_i \times \vec{e}_j}_{\epsilon_{ijk}\vec{e}_k} = (\underline{\omega}\times(\underline{x}-\underline{R}))^k\, \vec{e}_k$$

$$\Rightarrow \boxed{\dot{\tilde{x}}^i \vec{\tilde{e}}_i = (\underline{\dot{x}} - \underline{\dot{R}} - \underline{\omega}\times(\underline{x}-\underline{R}))^k\, \vec{e}_k}$$

$$\boxed{\frac{\tilde{d}}{\tilde{d}t}\vec{r} = \frac{d}{dt}\vec{r} - \frac{d}{dt}\vec{R} - \vec{\omega}\times(\vec{r}-\vec{R})}$$