Szám. Mó. Fiz. 1. 1. ZH. MINTAMEGOLDÁS (2014. Okt. 16.) —1—

1. a, $f'(x) = \left(x \sin(x^3)\right)' = \sin(x^3) + x \cdot \cos(x^3) \cdot 3x^2 = \underline{\underline{\sin(x^3) + 3x^3 \cos(x^3)}}$

b, $g(x) = \operatorname{tg} x \,; \qquad g(x_0) = g(0) = 0$

$g'(x) = \dfrac{1}{\cos^2 x} \,; \qquad g'(0) = 1$

$g''(x) = (-2)\cos^{-3} x \cdot (-\sin x) = 2 \sin x \cos^{-3} x \,; \qquad g''(0) = 0$

$g'''(x) = 2 \cos^{-2} x + 2 \sin x \cdot (-3) \cos^{-4} x (-\sin x) =$

$= 2 \cos^{-2} x + 6 \sin^2 x \cos^{-4} x \,; \qquad g'''(0) = 2$

Tehát $T_3(x) = \sum_{k=0}^{3} \dfrac{f^{(k)}(x_0)}{k!} x^k = 0 + \dfrac{1}{1!} x + \dfrac{0}{2!} x^2 + \dfrac{2}{3!} x^3 = \underline{\underline{x + \dfrac{x^3}{3}}}$

2, a, Kétszer parciálisan integrálunk:

$$\int_0^1 \underset{u}{x^2} \underset{v'}{e^x} \, dx = \underbrace{\left[x^2 e^x\right]_0^1}_{e-0} - \int_0^1 2x e^x \, dx = e - \Big(\underbrace{\left[2x e^x\right]_0^1}_{2e} - \int_0^1 2 e^x \, dx \Big) =$$

$u' = 2x \quad v = e^x$

$$= e - 2e + 2\int_0^1 e^x \, dx = -e + 2 \underbrace{\left[e^x\right]_0^1}_{e-1} = -e + 2(e-1) = \underline{\underline{e-2}}$$

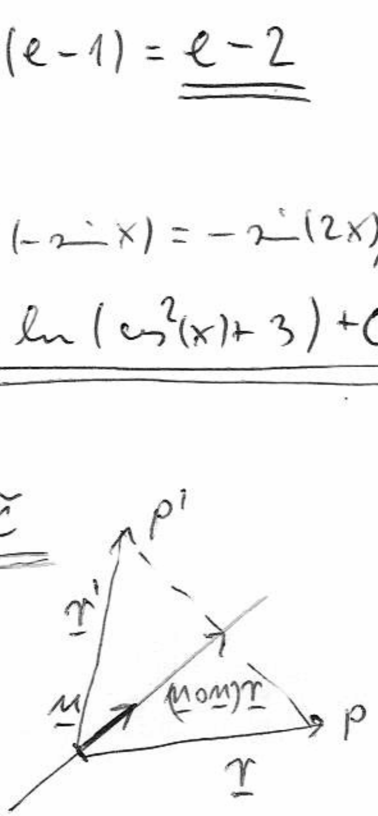
b, f'/f alakú lesz. $\left(\cos^2(x) + 3\right)' = 2\cos x(-\sin x) = -\sin(2x)$,

tehát $\displaystyle\int \frac{\sin(2x)}{\cos^2(x)+3} \, dx = -\int \frac{-\sin(2x)}{\cos^2(x)+3} \, dx = \underline{\underline{-\ln\left(\cos^2(x) + 3\right) + C}} =$

$\dfrac{f'}{f}$ alak

$= -\ln\left(\dfrac{1+\cos(2x)}{2} + 3\right) + C = \underline{\underline{-\ln\left(\cos(2x) + 7\right) + \tilde{C}}}$

3, $\underline{e} = \begin{bmatrix} 2 \\ 1 \end{bmatrix} ; \quad \underline{n} = \dfrac{\underline{e}}{\|\underline{e}\|} = \dfrac{1}{\sqrt{5}} \begin{bmatrix} 2 \\ 1 \end{bmatrix}$ egységvektor.



$\underline{\underline{T}} = \underline{n} \circ \underline{n} + (\underline{n} \circ \underline{n} - \underline{\underline{I}}) = 2 \underline{n} \circ \underline{n} - \underline{\underline{I}}$

$[\underline{\underline{T}}] = 2 \cdot \begin{bmatrix} 2 \\ 1 \end{bmatrix} \circ \begin{bmatrix} 2 \\ 1 \end{bmatrix} \cdot \dfrac{1}{5} - \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \dfrac{2}{5} \begin{bmatrix} 4 & 2 \\ 2 & 1 \end{bmatrix} - \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \underline{\underline{\dfrac{1}{5} \begin{bmatrix} 3 & 4 \\ 4 & -3 \end{bmatrix}}}$

4, $A(1,2,0)$; $B(-3,1,1)$; e: $\underline{r}(t) = \begin{bmatrix}2\\1\\5\end{bmatrix} + t\cdot \underbrace{\begin{bmatrix}2\\0\\1\end{bmatrix}}_{\underline{e}}$

A sík normálvektora:

$$\underline{n} = \overrightarrow{AB} \times \underline{e} = (\vec{B}-\vec{A}) \times \underline{e} = \begin{bmatrix}-4\\-1\\1\end{bmatrix} \times \begin{bmatrix}2\\0\\1\end{bmatrix} = \begin{bmatrix}-1\\6\\2\end{bmatrix}$$

Tehát a keresett sík egyenlete:

$$-x + 6y + 2z = \underbrace{-1 + 6\cdot 2 + 2\cdot 0}_{\text{átmegy } A\text{-n}} \Rightarrow \boxed{-x + 6y + 2z = 11}$$

5, i, Kifejtési tétellel:

$$((\underline{a} \times \underline{b}) \times \underline{a}) \cdot \underline{b} = (\underline{b} \cdot |\underline{a}|^2 - \underline{a}(\underline{a}\cdot\underline{b}))\cdot \underline{b} = |\underline{a}|^2 \cdot |\underline{b}|^2 - (\underline{a}\cdot\underline{b})^2$$

ii, Indexes számolással:

$$((\underline{a} \times \underline{b}) \times \underline{a}) \cdot \underline{b} = \varepsilon_{ijk}\, a_i\, b_j\, \varepsilon_{klm}\, a_l\, b_m =$$

$$= \varepsilon_{kij}\, \varepsilon_{klm}\, a_i\, b_j\, a_l\, b_m = (\delta_{il}\delta_{jm} - \delta_{im}\delta_{jl})\, a_i\, b_j\, a_l\, b_m =$$

$$= a_l^2\, b_j^2 - (a_i b_i)(a_j b_j) = |\underline{a}|^2 \cdot |\underline{b}|^2 - (\underline{a}\cdot\underline{b})^2 \checkmark$$

6, a, Legyen $p, q \in P$; $\alpha \in \mathbb{R}$. Ekkor $\alpha p + q \in P$, hiszen

$$(\alpha p + q)'(0) = \alpha p'(0) + q'(0) = \alpha 0 + 0 = 0 \checkmark$$

$$(\alpha p + q)(1) = \alpha p(1) + q(1) = \alpha 0 + 0 = 0 \checkmark$$

Tehát P zárt a lineáris kombinációra nézve.

b, A legfeljebb negyedfokú polinomok 5-dimenziós teret alkotnak, és mindkét feltétel eggyel csökkenti a dimenziót, így $\dim P = 5 - 1 - 1 = \underline{\underline{3}}$.

c, $\forall\, p \in P$ polinom tartalmazza az $(x-1)$ gyöktényezőt, és az x gyöktényezőt. Így

$b_1(x) = x(x-1)$; $b_2(x) = x^2(x-1)$; $b_3(x) = x^3(x-1)$ bázis P-ben.