1, a,
$$f(x)=(1+x^2)^x = e^{x\ln(1+x^2)}$$
$$f'(x)=e^{x\ln(1+x^2)}\cdot\left(x\ln(1+x^2)\right)' = \underline{\underline{(1+x^2)^x\left(\ln(1+x^2)+\frac{2x^2}{1+x^2}\right)}}$$

b,
$$g(x)=\ln(1+x) \qquad g(0)=\ln 1 = 0$$
$$g'(x)=\frac{1}{1+x} \qquad g'(0)=1$$
$$g''(x)=\frac{-1}{(1+x)^2} \qquad g''(0)=-1$$
$$g'''(x)=\frac{2}{(1+x)^3} \qquad g'''(0)=2$$

$$\left.\right\} \quad T_3(x)=x-\frac{1}{2!}x^2+\frac{2}{3!}x^3 = \underline{\underline{x-\frac{1}{2}x^2+\frac{1}{3}x^3}}$$

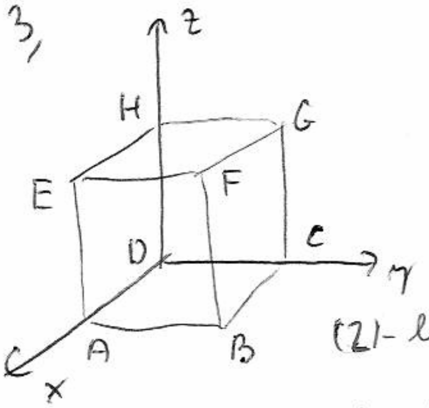
— ∘ —

2, a,
$$\int_0^1\sqrt{1-x^2}\,dx = \int_{t=0}^{\pi/2}\sqrt{1-\sin^2 t}\cdot\cos t\,dt = \int_{t=0}^{\pi/2}\cos^2 t\,dt = \int_0^{\pi/2}\frac{1+\cos(2t)}{2}\,dt =$$
$x=\sin t$
$dx=\cos t\,dt$
$$=\left[\frac{t}{2}+\frac{\sin(2t)}{4}\right]_0^{\pi/2} = \underline{\underline{\frac{\pi}{4}}}$$

b,
$$\int x\ln x\,dx = \frac{x^2}{2}\ln x-\int\frac{x^2}{2}\cdot\frac{1}{x}\,dx = \underline{\underline{\frac{x^2}{2}\ln x-\frac{x^2}{4}+C}}$$
u' v
$u=\frac{x^2}{2}$ $v'=\frac{1}{x}$

— ∘ —

3,



A keresett tr. legyen $\underline{\underline{A}}$, a standard bázisvektorok: $\underline{i}$, $\underline{j}$, $\underline{k}$

$E\xrightarrow{\underline{\underline{A}}}B:\ \underline{\underline{A}}(\underline{i}+\underline{k})=\underline{i}+\underline{j}$ (1)

$B\xrightarrow{\underline{\underline{A}}}C:\ \underline{\underline{A}}(\underline{i}+\underline{j})=\underline{j}$ (2)

$C\xrightarrow{\underline{\underline{A}}}G:\ \underline{\underline{A}}\,\underline{j}=\underline{j}+\underline{k}$ (3)

(2)-ből (3)-at kivonva: $\underline{\underline{A}}\,\underline{i}=-\underline{k}$ (4)

(1)-ből (4)-et –"– : $\underline{\underline{A}}\,\underline{k}=\underline{i}+\underline{j}+\underline{k}$ (5)

(4), (3) és (5) megadja $[\underline{\underline{A}}]$ oszlopait:

$$\underline{\underline{[\underline{\underline{A}}]=\begin{bmatrix}0&0&1\\0&1&1\\-1&1&1\end{bmatrix}}}$$

4, Az irányvektorok: $\underline{t}_e = \begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix}$; $\underline{t}_f = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$

A sík normálvektora: $\underline{\tilde{n}} = \underline{t}_e \times \underline{t}_f = \begin{bmatrix} -5 \\ -5 \\ 5 \end{bmatrix}$; $\underline{n} = \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}$

A sík egyenlete: $x + y - z = p$. (*)

e-nek egy pontja: $E(1, 2, -5)$; f-nek egy pontja: $F(3, 3, 6)$

Ezeket beírva (*) bal oldalába: $1 + 2 - (-5) = 8$; $3 + 3 - 6 = 0$

$\Longrightarrow p = \dfrac{8+0}{2} = 4$. Tehát a sík egyenlete: $\boxed{x + y - z = 4}$

5, i, Kifejtési tétellel:

$\underline{a} \cdot (\underline{a} \times (\underline{b} \times \underline{a})) = \underline{a} \cdot (\underline{b} \cdot |\underline{a}|^2 - (\underline{a} \cdot \underline{b}) \cdot \underline{a}) = (\underline{a} \cdot \underline{b}) \cdot |\underline{a}|^2 - (\underline{a} \cdot \underline{b}) \cdot |\underline{a}|^2 = 0$

ii, Indexesen:

$\underline{a} \cdot (\underline{a} \times (\underline{b} \times \underline{a})) = a_i \varepsilon_{ijk} a_j \varepsilon_{klm} b_l a_m = (\delta_{il} \delta_{jm} - \delta_{im} \delta_{jl}) a_i a_j b_l a_m =$

$= a_l a_m b_l a_m - a_m a_l b_l a_m = 0$

6, a, Legyen $A_1, A_2 \in M$, $\alpha \in \mathbb{R}$. Ekkor $\alpha A_1 + A_2 \in M$, hiszen

$(\alpha A_1 + A_2) B = \alpha A_1 B + A_2 B = \alpha B A_1 + B A_2 = B(\alpha A_1 + A_2)$

Tehát M zárt a lineáris kombinációképzésre.

b, Legyen $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \in M$.

$AB = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} a & 0 \\ c & 0 \end{bmatrix}$; $BA = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} a & b \\ 0 & 0 \end{bmatrix}$

Tehát $A \in M \Longleftrightarrow b = c = 0$. M általános eleme: $\begin{bmatrix} a & 0 \\ 0 & d \end{bmatrix}$

$a, d \in \mathbb{R}$.

$\underline{\underline{\dim M = 2}}$

c, Bázis M-ben: $\underline{\underline{\left\{ \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \right\}}}$