1, a, $f'(x)=\left(\frac{e^{-x^2}}{1+x^4}\right)'=\frac{-2x\,e^{-x^2}(1+x^4)-e^{-x^2}\cdot 4x^3}{(1+x^4)^2}$

b, $g(x)=\sqrt{1+x}$ $\qquad g(x_0)=1$

$g'(x)=\frac{1}{2}(1+x)^{-1/2}$ $\qquad g'(x_0)=\frac{1}{2}$

$g''(x)=-\frac{1}{4}(1+x)^{-3/2}$ $\qquad g''(x_0)=-\frac{1}{4}$

$g'''(x)=\frac{3}{8}(1+x)^{-5/2}$ $\qquad g'''(x_0)=\frac{3}{8}$

$$T_3(x)=\sum_{n=0}^{3}\frac{g^{(n)}(x_0)}{n!}(x-x_0)^n=\underline{\underline{1+\frac{1}{2}x-\frac{1}{8}x^2+\frac{1}{16}x^3}}$$

— o —

2, a, $\int x\sqrt{1+x^2}\,dx=\int \operatorname{sh}t\cdot\operatorname{ch}^2t\,dt\Big|_{t=\operatorname{arsh}x}=\frac{\operatorname{ch}^3t}{3}\Big|_{t=\operatorname{arsh}x}+C=$

$x=\operatorname{sh}t$

$dx=\operatorname{ch}t\,dt$

$$=\underline{\underline{\frac{1}{3}(1+x^2)^{3/2}}}+C$$

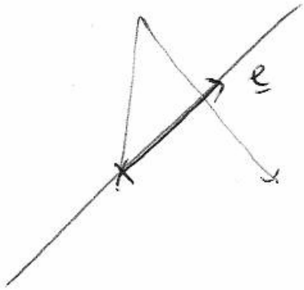
Vagy: $\frac{1}{2}\int \underbrace{2x(1+x^2)^{1/2}}_{f'\cdot f^{1/2}\text{ alak}}dx=\frac{1}{2}(1+x^2)^{3/2}\cdot\frac{2}{3}+C$

b, $\int_0^1 \underbrace{(2x+3)}_{u}\underbrace{e^{-x}}_{v'}dx=\left[(2x+3)(-e^{-x})\right]_0^1-\int_0^1 2\cdot(-e^{-x})\,dx=$

$u'=2 \qquad v=-e^{-x}$

$$=(-5e^{-1}+3)+2\left[-e^{-x}\right]_0^1=3-\frac{5}{e}-\frac{2}{e}+2=\underline{\underline{5-\frac{7}{e}}}$$

3, $\underline{e} = \begin{bmatrix} 1 \\ 2 \\ -2 \end{bmatrix}$; $|\underline{e}| = \sqrt{1+4+4} = 3$

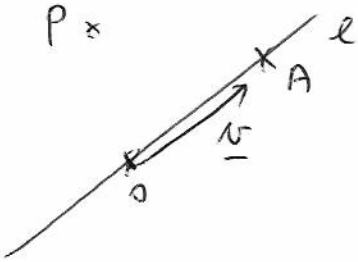


$\underline{\underline{T}} = \underline{e} \circ \underline{e} - (\underline{\underline{I}} - \underline{e} \circ \underline{e}) = 2\, \underline{e} \circ \underline{e} - \underline{\underline{I}}$

$[\underline{\underline{T}}] = 2 \cdot \frac{1}{9} \begin{bmatrix} 1 & 2 & -2 \\ 2 & 4 & -4 \\ -2 & -4 & +4 \end{bmatrix} - \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \frac{1}{9} \begin{bmatrix} -7 & 4 & -4 \\ 4 & -1 & -8 \\ -4 & -8 & -1 \end{bmatrix}$

4, $P(1,2,3)$; $e:\ x-3 = \frac{y-3}{2} = \frac{z-6}{3} = t$; $\underline{r}(t) = \begin{bmatrix} t+3 \\ 2t+3 \\ 3t+6 \end{bmatrix}$

Az egyenes irányvektora: $\underline{v} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$ Az egyenes egy pontja:



$\underline{r}(0) = \begin{bmatrix} 3 \\ 3 \\ 6 \end{bmatrix} = \underline{r}_A$

$\vec{AP} = \vec{P} - \vec{A} = \begin{bmatrix} -2 \\ -1 \\ -3 \end{bmatrix}$; $\vec{PA} = \begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix}$

A sík normálvektora: $\underline{n} = \underline{v} \times \vec{PA} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \times \begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix} = \begin{bmatrix} 6-3 \\ 6-3 \\ 1-4 \end{bmatrix} = \begin{bmatrix} 3 \\ 3 \\ -3 \end{bmatrix}$

A sík egyenlete: $\boxed{x + y - z = 0}$ átmegy P-n

5, $\underline{a} \cdot (\underline{a} \times (\underline{b} \times \underline{a})) = \underline{a} \cdot (\underline{b} \cdot |\underline{a}|^2 - \underline{a} \cdot (\underline{a} \cdot \underline{b})) = (\underline{a} \cdot \underline{b}) \cdot |\underline{a}|^2 - |\underline{a}|^2 (\underline{a} \cdot \underline{b}) =$

$= 0$

$\underline{a} \cdot (\underline{a} \times (\underline{b} \times \underline{a})) = a_i\, \varepsilon_{ijk}\, a_j\, \varepsilon_{klm}\, b_l\, a_m =$

$= (\delta_{il}\delta_{jm} - \delta_{im}\delta_{jl})\, a_i\, a_j\, b_l\, a_m = a_l\, a_m\, b_l\, a_m - a_m\, a_l\, b_l\, a_m = 0$

Látható, hogy $\underline{c} = \underline{a} \times (\underline{b} \times \underline{a}) \perp \underline{a}$, így $\underline{a} \cdot \underline{c} = 0$

6, a, $p_1, p_2 \in P$; $\alpha, \beta \in \mathbb{R}$ esetén

$$(A(\alpha p_1 + \beta p_2))(x) = (\alpha p_1 + \beta p_2)(x-1) = \alpha p_1(x-1) + \beta p_2(x-1) =$$

$$= \alpha (A p_1)(x) + \beta (A p_2)(x) \quad \forall x \in \mathbb{R} \text{ esetén, tehát}$$

$$A(\alpha p_1 + \beta p_2) = \alpha A p_1 + \beta A p_2$$ ✓

b, Legyen $b_k(x) = x^k$, $k = 0, 1, 2, 3$.

$$(A b_0)x = b_0(x-1) = 1$$

$$(A b_1)(x) = b_1(x-1) = x-1 = (b_1 - b_0)(x)$$

$$(A b_2)(x) = b_2(x-1) = (x-1)^2 = x^2 - 2x + 1 =$$

$$= (b_2 - 2b_1 + b_0)(x)$$

$$(A b_3)(x) = b_3(x-1) = (x-1)^3 = x^3 - 3x^2 + 3x - 1 =$$

$$= (b_3 - 3b_2 + 3b_1 - b_0)(x)$$

$$[A] = \begin{bmatrix} 1 & -1 & 1 & -1 \\ 0 & 1 & -2 & 3 \\ 0 & 0 & 1 & -3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$