BORN APPROXIMATION IN ONE DIMENSION

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Using Griffiths's Green's function for the one-dimensional Schrödinger equation:

$$\psi(x) = \psi_0(x) - \frac{im}{\hbar^2 k}\int_{-\infty}^{\infty} e^{ik|x-x_0|} V(x_0)\,\psi(x_0)\,dx_0 \tag{1}$$

we can work out the Born approximation in the one-dimensional case. The idea is we replace $\psi(x_0)$ inside the integral by the incident plane wave form $\psi_0(x_0)$. Assuming that the potential is zero outside a finite distance from the origin, we want the wave function in two regions: $x \ll 0$ and $x \gg 0$. The former will give the reflected wave and the latter the transmitted wave.

For $x \ll 0$ (and at a distance where $V(x) = 0$) we have

$$e^{ik|x-x_0|} = e^{-ikx} e^{ikx_0} \tag{2}$$

$$\psi_0(x_0) = Ae^{ikx_0} \tag{3}$$

where A is the normalization constant. The Born approximation for this region is

$$\psi(x) = Ae^{ikx} - \frac{im}{\hbar^2 k} e^{-ikx}\int_{-\infty}^{\infty} e^{ikx_0} V(x_0)\,Ae^{ikx_0}\,dx_0 \tag{4}$$

$$= A\left[e^{ikx} - \frac{im}{\hbar^2 k} e^{-ikx}\int_{-\infty}^{\infty} e^{2ikx_0} V(x_0)\,dx_0\right] \tag{5}$$

The scattering amplitude and reflection coefficient for the reflected particle are therefore

$$f_R = -\frac{im}{\hbar^2 k}\int_{-\infty}^{\infty} e^{2ikx_0} V(x_0)\,dx_0 \tag{6}$$

$$R = |f_R|^2 \tag{7}$$

$$= \left(\frac{m}{\hbar^2 k}\right)^2 \left|\int_{-\infty}^{\infty} e^{2ikx_0} V(x_0)\,dx_0\right|^2 \tag{8}$$

[Note that we're assuming that $x<x_0$ so although the limits of the integral are infinite, we're implicitly assuming that $V=0$ for all $x_0>x$ so the integral isn't really over an infinite range.]

For $x\gg0$ we have

$$e^{ik|x-x_0|}=e^{ikx}e^{-ikx_0} \tag{9}$$

$$\psi_0(x_0)=Ae^{ikx_0} \tag{10}$$

The Born approximation here is

$$\psi(x)=Ae^{ikx}-\frac{im}{\hbar^2k}e^{ikx}\int_{-\infty}^{\infty}e^{-ikx_0}V(x_0)Ae^{ikx_0}dx_0 \tag{11}$$

$$=Ae^{ikx}\left[1-\frac{im}{\hbar^2k}\int_{-\infty}^{\infty}V(x_0)\,dx_0\right] \tag{12}$$

The scattering amplitude and transmission coefficient are therefore

$$f_T=1-\frac{im}{\hbar^2k}\int_{-\infty}^{\infty}V(x_0)\,dx_0 \tag{13}$$

$$T=|f_T|^2 \tag{14}$$

$$=1+\left(\frac{m}{\hbar^2k}\right)^2\left|\int_{-\infty}^{\infty}V(x_0)\,dx_0\right|^2 \tag{15}$$

The Born approximation fails for transmission in this case, since $T>1$ which is impossible. We can still get an estimate of T from $T=1-R$.

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