Simulations in Statistical Physics

Course for MSc physics students

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Algorithmically defined models

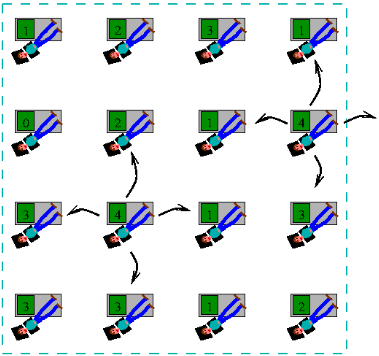
- Self-Organized Criticality
  - Bak-Tang-Wiesenfeld model
  - Forest fire model
  - Bak-Sneppen model of evolution
- Traffic models
- 1d driven systems

Self-Organized Criticality

- Critical state: inflection point in the critical isotherm
- Power law functions of correlation length, relaxation time
- Control parameter, generally temperature
- Critical point as an attractor?
- Why? Power law: We see many cases
  - $1/f$ noise (music, ocean, earthquakes, flames)
  - Lack of scales (market, earthquakes)
- Underlying mechanism?

Bak-Tang-Wiesenfeld model

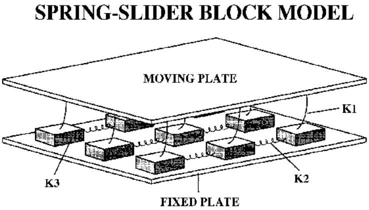
- Originally a sandpile model
- Better explained as a *Lazy Bureaucrat model*:
  - Bureaucrats are sitting in a large office in a square lattice arrangement
  - Occasionally the boss comes with a dossier and places it on a random table
  - The bureaucrats do *nothing* until they have less than 4 dossiers on their table



- Once a bureaucrat has 4 or more dossiers on its table starts to panic and distributes its dossiers to its 4 neighbors
- The ones sitting at the windows give also 1 dossier to its neighbors and throw the rest out of the window.

Bak-Tang-Wiesenfeld model

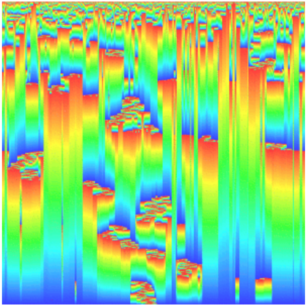
- Originally a sandpile model
- Better explained as a *Lazy Bureaucrat model*:
- Best application: Spring block model of earthquakes:
  - Masses sitting on a frictional plane in a grid are connected with springs to eachother and to the top plate



- Top plate moves slowly, increasing the stress on the top springs slowly and randomly
- If force is large enough masses move which increases the stress on the neighboring masses

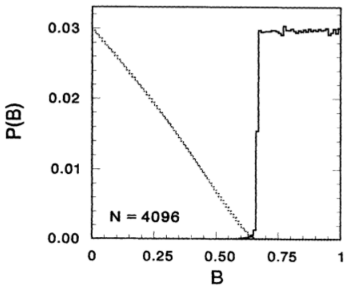
Bak-Sneppen model of evolution

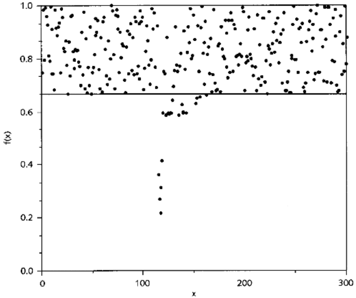
- N species all depends on two other (ring geometry)
- Each species are characterized by a single *fitness*
- In each turn the species with the lowest fitness dies out and with it its two neighbors irrespective of their fitness
- These 3 species are replaced by new ones with random fitness
- Inital and update fitness is uniform between $[0, 1]$



Bak-Sneppen model of evolution: Results

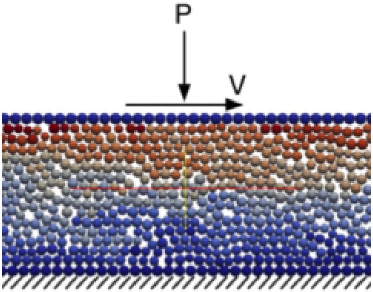
- Steady state with avalanches
- Avalanches start with a fitness $f > f_c \simeq 0.66$
- Avalanche size distribution power law
- Distance correlation power law

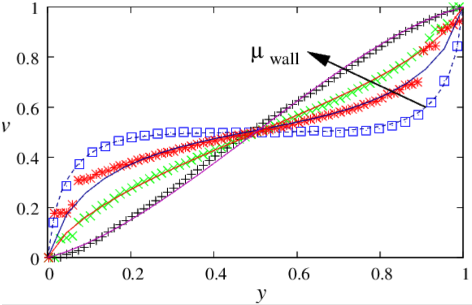




Bak-Sneppen model of evolution an application: Granular shear

- Fitness $\rightarrow$ Effective friction coefficient
- Speciment with lowest fitness dies out $\rightarrow$ block is sheared at weakest position (shear band)
- Neighbors, related species die out and replaced by new species $\rightarrow$ structure gets random aroung the shear band.

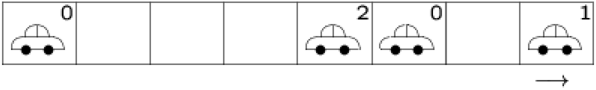




Traffic models

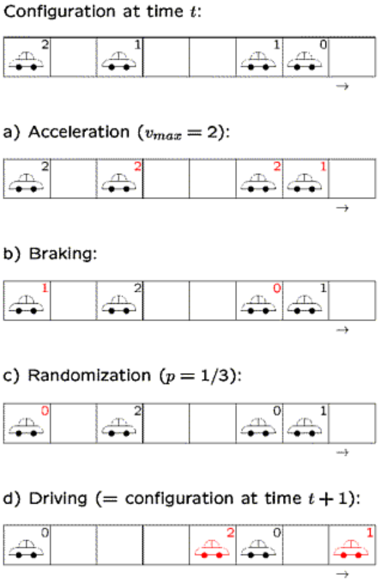
Nagel–Schreckenberg model

- Periodic 1d lattice (ring) Autobahn
- discretized in space and time
- Cars occupying a lattice moving with velocities $v = 0, 1, 2, 3, 4, 5$
- Remark, if max speed is 126 km/h, then lattice length is 7 m, a very good guess for a car in a traffic jam
- It uses parallel update: at each timestep all cars move v sites forward

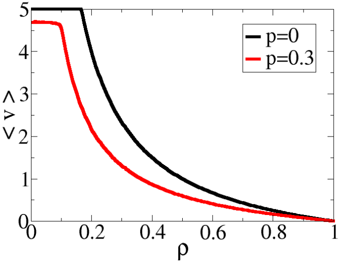


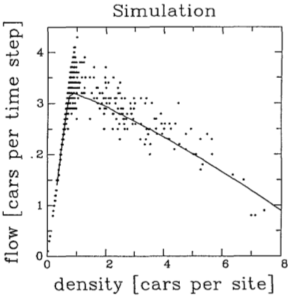
Nagel–Schreckenberg model

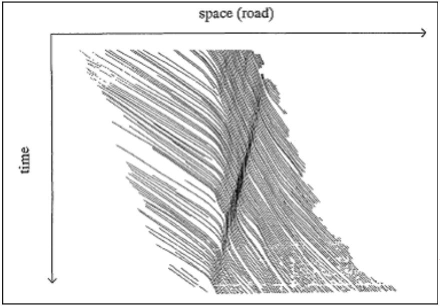
- Algorithm:
  1. **Acceleration**: All cars not at the maximum velocity increase their velocity by 1
  2. **Slowing down**: Speed is reduced to distance ahead (1 sec rule)
  3. **Randomization**: With probability *p* speed is reduced by 1
  4. **Car motion**: Each car moves forward the number of cells equal to their velocity.

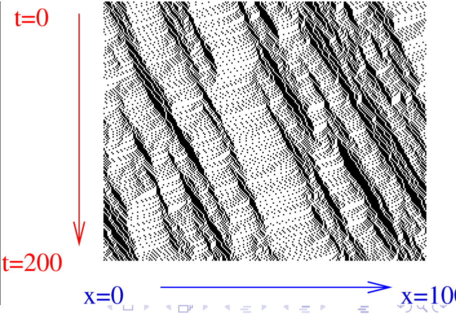


Emergence of traffic jams







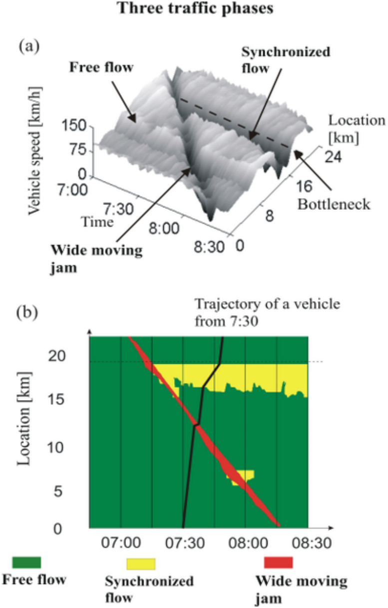


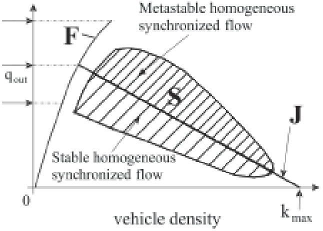
Nagel–Schreckenberg model

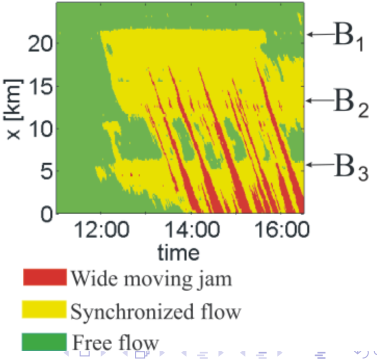
- Transition from free-flow to jammed state
- Jammed state is a phase-separated phase
- Without randomization a sharp transition
- Used in NRW to predict traffic jams

autobahn.NRW
Hochrechnung für Bundesautobahnen
Montag, 01.07.2013, 08:05 h
aktuelle Verkehrslage
Prognose 30 min
Prognose 60 min
Langzeit Prognose
Fahrzeitprognose
Farbmodus: Standard
Baustelleninfo
Verkehrsdichten
frei
zähfließend
dicht
gestaut
Baustelle mit geringer Staugefahr
Baustelle mit hoher Staugefahr
Anschlussstelle
Sperrung der Autobahn
Vollsperrung der Anschlussstelle
Teilsperrung der Anschlussstelle
Informationen
Impressum
Version 3.6.1
0 km
15 km
Wesel
Recklinghausen
Lünen
Geldern
Kamp-Lintfort
Moers
Bottrop
Gelsenk.
Herne
Dortmund
Oberhsn.
Essen
Bochum
Duisburg
Mülheim
Witten
Kempen
Krefeld
Hagen
Velbert
Viersen
Gevelsberg
Düsseldorf
Mönchengladbach
Neuss
Wuppertal
Lüdenscheid

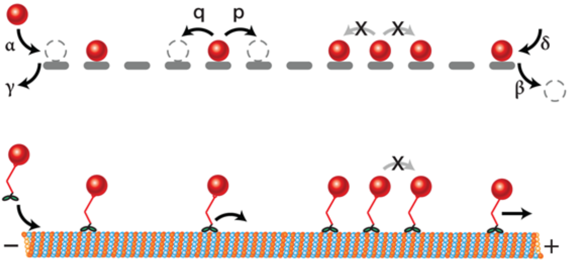
Three-phase traffic theory





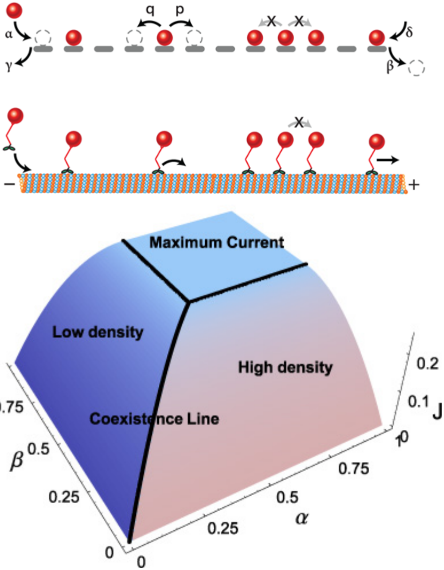


Asymmetric simple exclusion process



- $p + q = 1$
- If $p = q$ then SEP a Markov-process
- Generally $\gamma = \delta = 0$
- α and β determines the phase diagram

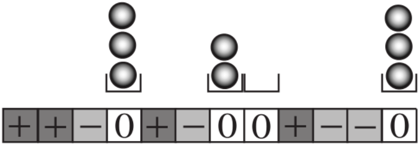
Asymmetric simple exclusion process



Three state ASEP

$$+\ 0 \xrightarrow{\alpha} 0\ + \quad ; \quad 0\ - \xrightarrow{\alpha} -\ 0 \quad ; \quad +\ - \underset{q}{\overset{1}{\rightleftarrows}} -\ +$$

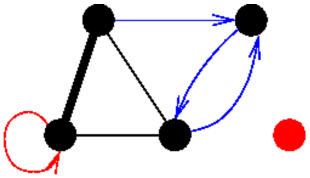
- If q small three blocks $(00\ldots00++\cdots++--\cdots--)$
- Mixed state above $q=1$
- Numerical simulations suggested an other phase transition at $q_c < 1$
- Actually false, only correlation length is finite but large $\sim \mathcal{O}(10^{70})$
- Correspondence to Zero Range Process



Networks

Complex networks

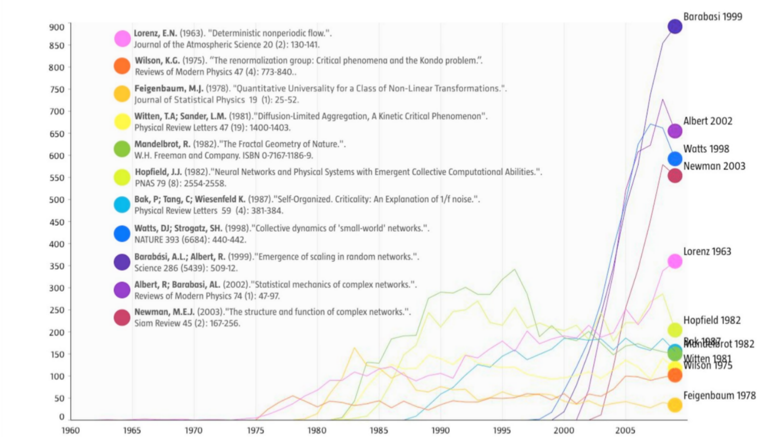
- Mathematics: Graphs
- Vertices, nodes, points
- Edges, links, arcs, lines
  - Directed or undirected
  - Loop
  - Multigraph
  - Wighted graphs
  - Connected



Complex networks

Phenomenon	Nodes	Links
Ising	Spins	Interaction(neighbors)
Cell metabolism	Molecules	Chem. reactions
Sci. collaboration	Scientists	Joint papers
WWW	Pages	URL links
Air traffic	Airports	Airline connections
Economy	Firms	Trading
Language	Words	Joint appearance

Complex networks, citations



Random Networks

Generate networks:

- From data:
  - Phone calls
  - WWW links
  - Biology database
  - Air traffic data
  - Trading data
- Generate randomly
  - From regular lattice by random algorithm (e.g. percolation)
  - Erdős-Rényi graph
  - Configurations model
  - Barabási-Albert model

Erdős-Rényi

- P. Erdős, A. Rényi, *On random graphs*, Publicationes Mathematicae Debrecen, Vol. 6 (1959), pp. 290-297 (cit 789)
- Two variants:
  1. $G(N, M)$: N nodes, M links
  2. $G(N, P)$: N nodes, links with p probability (all considered)
- Algorithm
  1. $G(N, M)$:
     - Choose i and j randomly $i, j \in [1, N]$ and $i \neq j$
     - If there is no link between i an j establish one
  2. $G(N, P)$: (Like percolation)
     - Take all $\{i, j\}$ pairs $(i \neq j)$
     - With probability p establish link between i and j

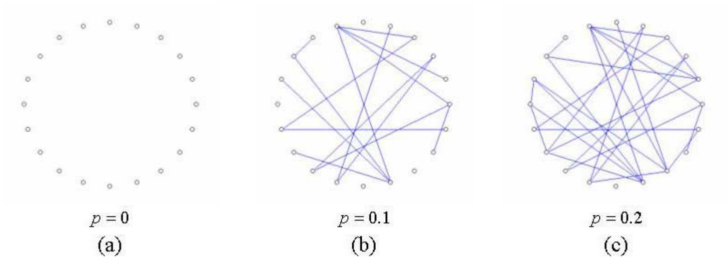
Erdős-Rényi

- Degree distribution

$$P(k) = \binom{N-1}{k} p^k (1-p)^{N-1-k}$$

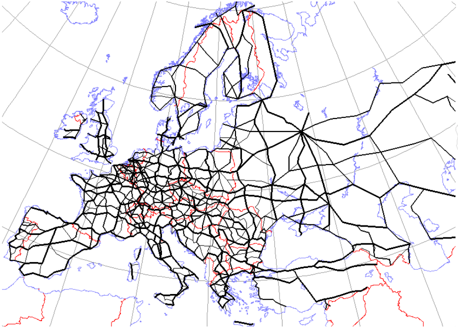
- For large N and Np =const it is a Poisson distribution

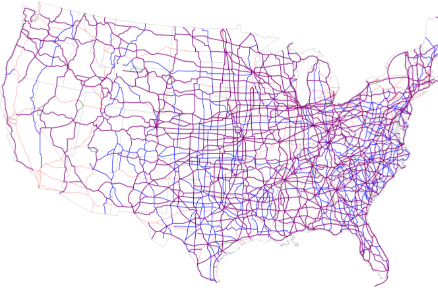
$$P(k) \to \frac{(np)^k e^{-np}}{k!}$$



Erdős-Rényi

- Real life: Read networks

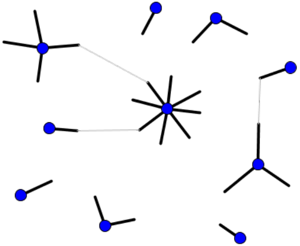


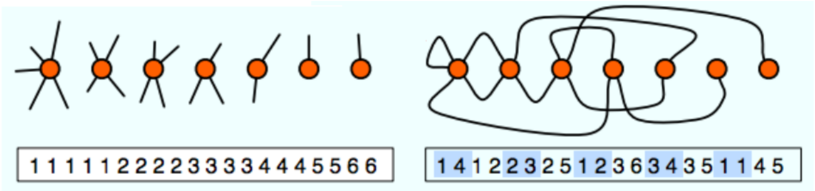


Most networks are different!

Configuration model

- Get the nodes ready with desired degree distribution
- Connect them randomly
- Self loops, and multiple links are created
- Problems at the end

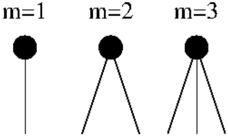




Preferential attachment

Barabási-Albert graph

- Initially a fully connected graph of m_0 nodes
- All new nodes come with m links ($m \leq m_0$)



- Links are attached to existing nodes with probability proportional to its number of links
- k_i is the number links of node i, then

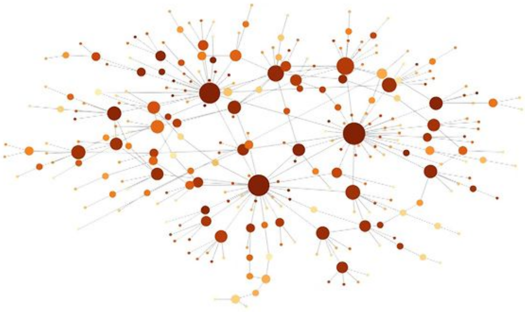
$$p_a = \frac{k_i}{\sum_j k_j}$$

Barabási-Albert graph

- Degree distribution

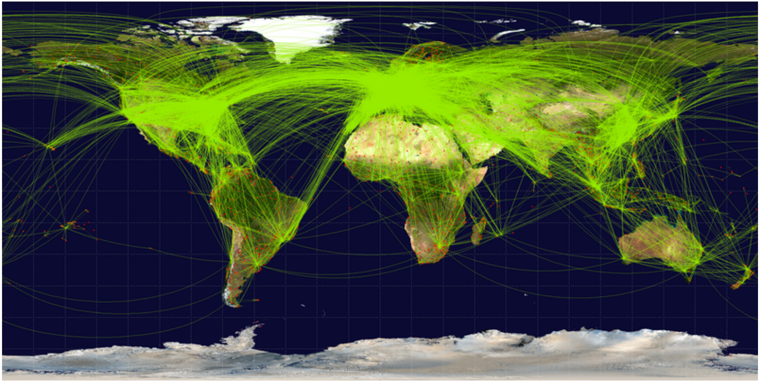
$$p(k) \sim k^{-3}$$

- Independent of m!

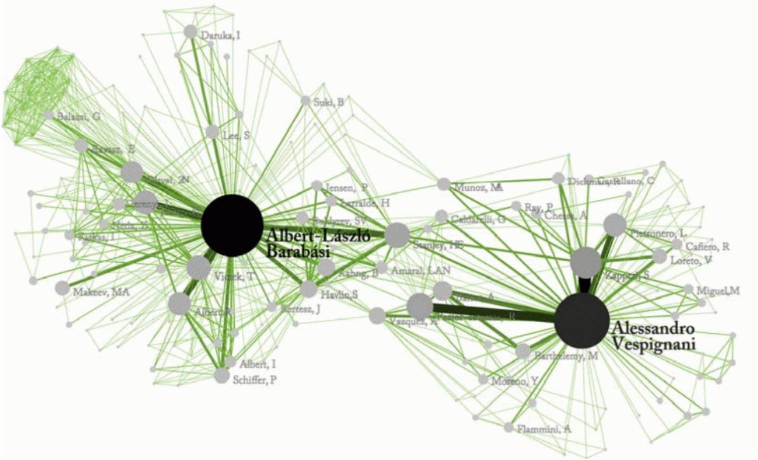


$m = 1$

Scalefree network example: Flight routes



Scalefree network example: Co-authorship

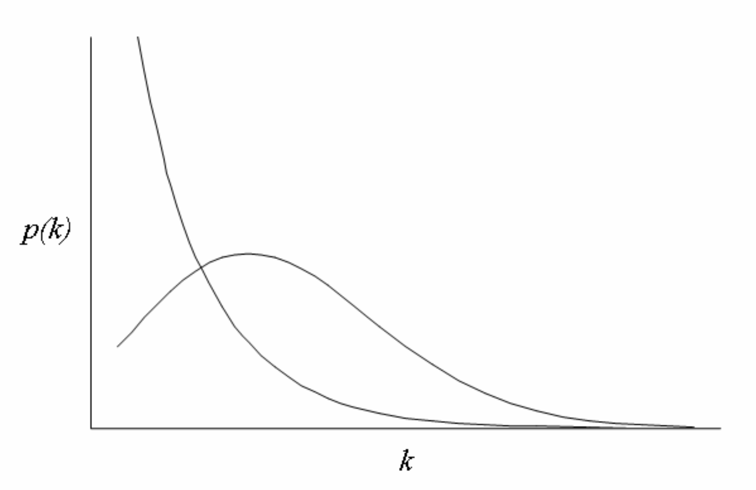


Algorithm for Barabási-Albert graph

1. $n = m_0$ number of existing nodes
2. $K = \sum_j k_j$ total number of connections
3. r random number $r \in [0, K]$
4. Find $i_{\max}$ for which $\sum_{j=0}^{i_{\max}} k_j < r$
5. If there is no edge then add one between nodes $n + 1$ and $i_{\max}$
6. If node $n + 1$ has less than m connections go to 3.
7. Increase n by 1
8. If $n < N$ go to 2.

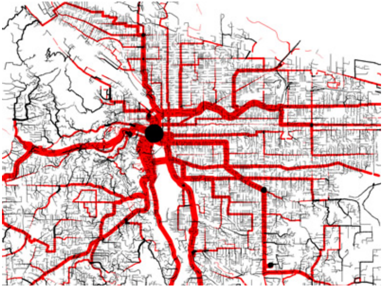
Properties of networks

- Degree distribution



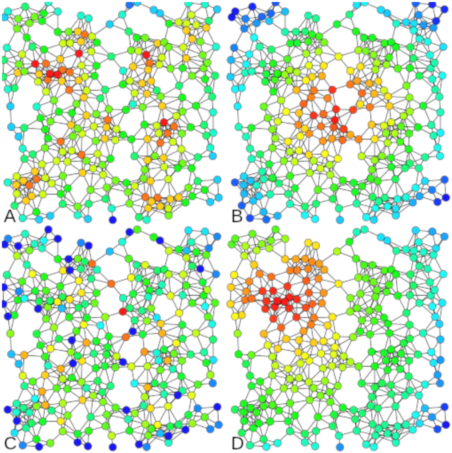
Properties of networks

- Degree distribution
- Shortest path

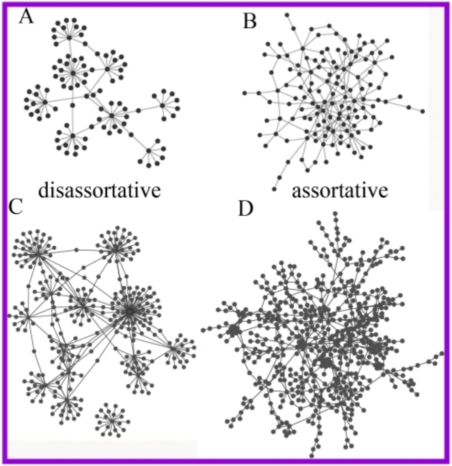
Properties of networks

- Degree distribution
- Shortest path
- Centrality
  - Degree centrality
  - Closeness centrality
  - Betweenness centrality
  - Eigenvector centrality (Page rank)



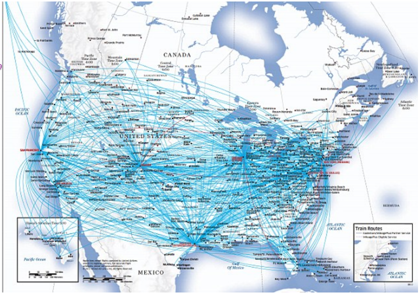
Properties of networks

- Degree distribution
- Shortest path
- Centrality
- Assortativity



Percolation and attack on random networks

- Failure: equivalent to percolation: remove nodes at random
- Attack: remove most connected nodes



Percolation and attack on random networks

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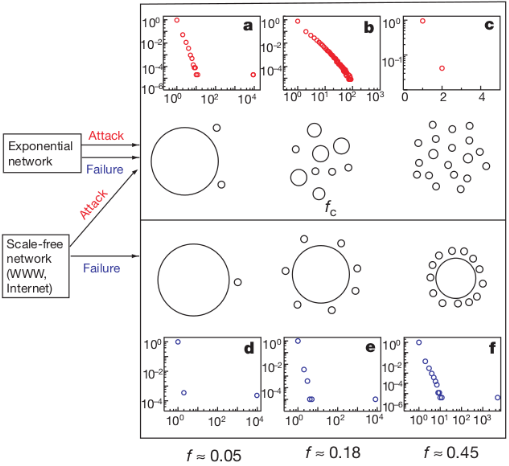
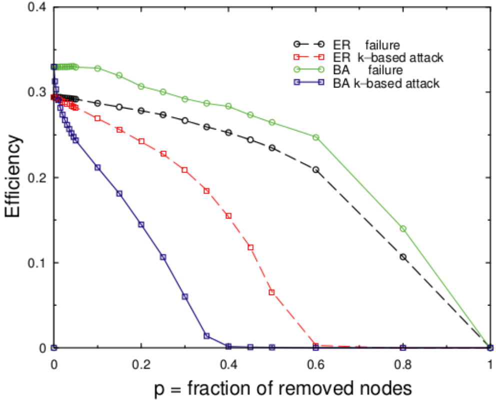


Percolation and attack on random networks

- Efficiency:

$$E(G) = \frac{1}{N(N-1)} \sum_{i \neq j} \frac{1}{t_{ij}}$$

  t_{ij} the shortest path between i and j.
- $N = 2000$, $k = 10^4$



Percolation and attack on random networks