Kvantummechanika

1.

$$w_v d\upsilon = 2k_B T \frac{\pi}{c^3} \upsilon^2 d\upsilon$$

3.

$$\left\langle x \right\rangle_\varphi = \int_{-\infty}^{\infty} |a(x')|^2 \; x dx' = \int_{-\infty}^{\infty} \underbrace{\left| \left\langle \chi(x,x') \middle| \varphi(x) \right\rangle \right|^2}_{P(x')} x' dx'$$

4.

$$\frac{i}{\hbar} \left\langle \Psi \middle| [\hat{H}, \hat{A}] \middle| \Psi \right\rangle = \frac{d}{dt} < A >_\Psi \qquad \frac{d}{dt} < x >_\Psi = \frac{i}{\hbar} \left\langle \Psi \middle| -\frac{\hbar^2}{2m} \frac{\partial}{\partial x} \middle| \psi \right\rangle$$

$$\frac{d}{dt} < \hat{p}_x >_\Psi = < -\frac{dV}{dx} >_\Psi$$

6.

$$\hat{x} = \sigma(\hat{a} + \hat{a}^+), \qquad p = \frac{\hbar}{2i\sigma}(\hat{a} - \hat{a}^+), \quad \sigma = \sqrt{\frac{p}{2m\omega}} \quad \hat{a} = \frac{1}{2\sigma}\hat{x} + \frac{i}{\hbar}\sigma\hat{p}, \quad \hat{a}^+ = \frac{1}{2\sigma}\hat{x} - \frac{i}{\hbar}\sigma\hat{p}$$

$$\hat{N} = \hat{a}^+ \hat{a}$$

$$\hat{L}_\pm = \hat{L}_x \pm i\hat{L}_y \qquad \hat{L}_z \hat{L}_\pm = \hat{L}_x(\hat{L}_z \pm \hbar) \pm i\hat{L}_y(\hat{L}_y \pm \hbar) \qquad \hat{L}_\pm \hat{L}_\mp = \hat{L}^2 - \hat{L}_z^2 \pm \hbar \hat{L}_z$$

$$|I| = \frac{L|-e|}{2m\pi r^2} \qquad \hat{\mu}_L = \frac{e\hbar}{2m}\sqrt{l(l+1)}$$

7.

$$\Delta_{r,\vartheta,\varphi} = \Delta_r + \frac{1}{r^2}\Delta_{\vartheta,\varphi} \qquad \hat{L}^2 = -\hbar^2 \Delta_{\vartheta,\varphi} \to \frac{1}{\sin^2\vartheta}\frac{\partial}{\partial\vartheta}\left(m\vartheta \frac{\partial}{\partial\vartheta} \right) + \frac{1}{m^2\vartheta}\frac{\partial^2}{\partial\varphi^2}$$

$$\Delta_r R = \frac{1}{r^2} \underbrace{\frac{d}{dr}\left(r^2 \frac{dR}{dr} \right)}_{2rR' + r^2 R''} \qquad -\frac{\hbar}{2m}\left[\Delta_r R\right] + V_{eff}(r) R = ER$$

$$V_{eff} = \frac{L^2}{2mr^2} - \frac{e^2}{4\pi\varepsilon_0}\frac{1}{r}$$

8.

$$\hat{\hat{s}} \times \hat{\hat{s}} = i\hbar\hat{\hat{s}} \qquad \sigma_y = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix} \quad \sigma_x = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \quad \sigma_z = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

9.

$$\left| \varphi_k \right\rangle = \left| \psi_n \right\rangle + \lambda \left| \varphi_{1k} \right\rangle + \lambda^2 \left| \varphi_{2k} \right\rangle + ... \qquad \varepsilon_k = E_k + \lambda\varepsilon_{1k} + \lambda^2\varepsilon_{2k} + ...$$

$$\hat{H}_0|\varphi_{1k}\rangle + \hat{H}'|\psi_n\rangle = E_k|\varphi_{1n}\rangle + \varepsilon_{1k}|\psi_n\rangle \qquad \langle\psi_k|\hat{H}'|\psi_k\rangle = \varepsilon_{1k}$$

$$\left. a_{kl} = \frac{\langle\psi_l|\hat{H}'|\psi_k\rangle}{E_k - E_l}\right]_{k\neq l} \qquad |\varphi_k\rangle = |\psi_k\rangle + \sum_{k\neq l}\frac{\langle\psi_l|\hat{H}'|\psi_k\rangle}{E_k - E_l}|\psi_l\rangle$$

$$\hat{H} = \hat{H}_0 + \frac{eB}{2m}\hat{L}_z \qquad \underbrace{(E_n + \hbar\omega_L m_l)}_{\varepsilon(n,m,\omega_L)}\Psi_n = \varepsilon\Psi_n$$

$$\varepsilon_k = E_k + \langle\psi_k|\hat{H}'|\psi_k\rangle + \sum_{k\neq l}\frac{\left|\langle\psi_k|\hat{H}'|\psi_l\rangle\right|^2}{E_k - E_l}$$

$$\sum_{l=1}^{d}\left[c_{kl}\underbrace{\langle\psi_{kl'}|\hat{H}_0|\psi_{kl}\rangle}_{E_k\delta_{ll'}} + c_{kl}\underbrace{\langle\psi_{kl'}|\hat{H}'|\psi_{kl}\rangle}_{=\hat{H}'_{ll'}}\right] = \varepsilon_k\left(\underbrace{\sum_{l=1}^{d}c_{kl}\langle\psi_{kl'}|\psi_{kl}\rangle}_{c_{kl'}}\right)$$

10.

$$\sum_k\left\{\frac{\hbar}{i}\dot{a}_k - a_k\hat{H}'\right\}|\Psi_k\rangle e^{-i\frac{E_k}{\hbar}t} = 0 \qquad \frac{\hbar}{i}\dot{a}_l + \sum_k H'_{lk}a_k\exp(i\omega_{lk}t) = 0$$

$$a_l(t) - a_l(0) = -\frac{i}{\hbar}\sum_k^{\infty}\int_0^t a_k(t'')H'_{kl}(t'')\exp(i\omega_{kl}t'')dt''$$

$$\{a_l(0) = a_l^{(0)}\} \to a_l^{(1)}(t) = -\frac{i}{\hbar}\sum_k^{\infty}\int_0^t a_l^{(0)}H'_{kl}(t'')\exp(i\omega_{kl}t'')dt''$$

$$a_v^{(0)} = \delta_{kv} - \frac{i}{\hbar}\underbrace{\int_0^t H'_{kv}(t'')e^{i\omega_{kv}t''}dt''}_{Fourier} \qquad a_v^{(1)} \underset{v\neq k}{=} -i\frac{2\pi}{\hbar}\tilde{H}'_{kv}(\omega_{vk})$$

$$P(E_k \to E_v) = \frac{4\pi^2}{\hbar^2}\left|\langle\Psi_v|H'_0|\Psi_k\rangle\right|^2|W(\omega_{kv})|^2$$

$$\tilde{W}(\omega) = \frac{1}{2\pi}\int_0^T \sin(\omega_0 t'')e^{i\omega t''}dt'' = \frac{1}{4\pi}\left[\frac{1 - e^{i(\omega_0-\omega)t}}{\omega_0 + \omega} - \frac{1 - e^{i(\omega_0-\omega)t}}{\omega - \omega_0}\right]$$

$$\left|\tilde{W}(\omega)\right|^2 = \frac{1}{4\pi}\left[\frac{\sin\frac{\omega_0 \pm \omega}{2}T}{\frac{\omega_0 \pm \omega}{2}}\right]^2$$

12.

$$\phi_A(12) = \underbrace{\Psi^0_{ASZ}(12)}_{\frac{1}{\sqrt{2}}\begin{bmatrix}\Psi_1^0(1) & \Psi_1^0(2)\\ \Psi_2^0(1) & \Psi_2^0(2)\end{bmatrix}}\alpha(1)\alpha(2) \qquad \phi_B(12) = \Psi^0_{ASZ}(12)\beta(1)\beta(2) \to \chi_{SZ}(12)$$

$$\phi_{C'} = \frac{1}{\sqrt{2}} \begin{bmatrix} \Psi_1^0(1)\alpha(1) & \Psi_1^0(2)\alpha(2) \\ \Psi_2^0(1)\beta(1) & \Psi_2^0(2)\beta(2) \end{bmatrix} \quad \phi_{D'} = \frac{1}{\sqrt{2}} \begin{bmatrix} \Psi_1^0(1)\beta(1) & \Psi_1^0(2)\beta(2) \\ \Psi_2^0(1)\alpha(1) & \Psi_2^0(2)\alpha(2) \end{bmatrix}$$

$$\phi_C = \frac{1}{\sqrt{2}}(\phi_{C'} + \phi_{D'}) = \ldots = \overbrace{\Psi_{ASZ}^0(12)}^{ASZ} \overbrace{\frac{2}{\sqrt{2}}\left(\alpha(1)\beta(2) + \alpha(2)\beta(1)\right)}^{\chi_{SZ}(12)}$$

$$\phi_D = \frac{1}{\sqrt{2}}(\phi_{C'} - \phi_{D'}) = \ldots = \frac{1}{\sqrt{2}}\left(\Psi_1^0(1)\Psi_2^0(2) + \Psi_1^0(2)\Psi_2^0(1)\right) \underbrace{\frac{1}{2}\begin{bmatrix} \alpha(1) & \alpha(2) \\ \beta(1) & \beta(2) \end{bmatrix}}_{\chi_{ASZ}(12)}$$

$$E_x = \int dV_1 \int dV_2 \frac{\Psi_1^{0^*}(1)\Psi_2^0(1)\Psi_1^{0^*}(2)\Psi_2^0(2)}{4\pi\varepsilon_0 r_{12}} e^2$$

$$\left(-\frac{\hbar^2}{2m}\Delta_1 - \frac{2e^2}{4\pi\varepsilon_0}\frac{1}{r_1} + U_2(1)\right)\Psi_1(1) = E_1\Psi_1(1) \quad \left(-\frac{\hbar^2}{2m}\Delta_2 - \frac{2e^2}{4\pi\varepsilon_0}\frac{1}{r_2} + U_1(2)\right)\Psi_2(2) = E_2\Psi_2(2)$$

$$\rho_{c2} = -e|\varphi_2(2)|^2 \quad U_{c2}(1) = \int dV_2 \frac{e|\Psi_2(2)|^2}{4\pi\varepsilon_0 r_{12}} \quad U_{x2}(1)\Psi_1(1) = -\int dV_2 \frac{e^2\Psi_2^*(2)\Psi_1(2)\Psi_2(1)}{4\pi\varepsilon_0 r_{12}}$$

14.

$$\underline{j} = \frac{\hbar}{2mi}(\Psi^*\nabla\Psi - (\nabla\Psi)^*\Psi)$$

$$T^G = \left|\frac{\Psi_2}{\Psi_1}\right|^2 \qquad T^G = \left|\frac{Ae^{-\alpha\cdot x_L}}{Ae^{-\alpha\cdot x_1}}\right|^2 \approx e^{-2\alpha(x_L - x_1)} = \exp\left(-2\sqrt{\frac{2m}{\hbar}}\sqrt{V_0 - E}(x_L - x_1)\right)$$

$$T = \frac{1}{1 + \frac{V_0^2}{4E(V_0 - E)} sh^2\left(\sqrt{\frac{2mL^2}{\hbar^2}(V_0 - E)}\right)}$$

$$\Psi(\underline{r}) = e^{ikz} + f(\vartheta)\frac{e^{ik}}{V} \qquad J = \frac{\hbar k}{m}\frac{|f|^2}{r^2} \qquad \sigma_\vartheta = \frac{\hbar k}{m}|f(\vartheta)|^2 \qquad L = \vartheta br$$

Statisztikus fizika

1.

$$V_d(r) = \frac{\pi^{d/2}}{\Gamma(d/2+1)} r^d$$

2.

$$f(E_1) \sim \exp\left(-\frac{E_1 - \tilde{E}_1}{2\Delta^2}\right) \qquad \frac{1}{\Delta^2} = -\left(\frac{\partial^2 "S_1"}{\partial E_1^2} + \frac{\partial^2 "S_2"}{\partial E_2^2}\right) \approx \frac{a}{N_1} + \frac{b}{N_2}$$

3.

$$\frac{k_B}{\varepsilon_o} \ln \frac{N_-}{N_+} = \frac{1}{"T"} \qquad \ln \Omega_o = \frac{5N}{2} + \frac{3N}{2} \ln\left[\frac{2E}{3N} 2m\pi \frac{1}{h^2}\left(\frac{V}{N}\right)^{2/3}\right]$$

4.

$$\ln \rho(q,p) = const + \ln \Omega'(E_R) + \left(\frac{\partial \ln \Omega'(E)}{\partial E}\right)_{E_R} \left(-E(q,p)\right) + O\left(\frac{1}{N_R^2}\right) O(N^2)$$

$$\int_{-\infty}^{\infty} e^{-x^2} dx = \sqrt{\pi} \qquad Z_1 = \frac{k_B T}{\hbar \omega}$$

$$P(E) = const \cdot \exp\left(-\frac{(E - \tilde{E})^2}{2 k_B T^2 C_V}\right)$$

5.

$$\ln \rho_N = konst + \left|\frac{\partial \ln \Omega'_N(E)}{\partial E}\right|_{ER,NR} \left(-E_N(q,p)\right) + \left|\frac{\partial \ln \Omega'_N(E)}{\partial N}\right|_{ER,NR} (-N)$$

$$Z_{Nid} = \left(\frac{V}{h^3} \sqrt[3]{2\pi m k_B T}\right)^N \frac{1}{N!} \qquad \phi_{idgáz} = k_B T \left\{ e^{\beta\mu} \frac{V}{h^3} \sqrt{2\pi m k_B T} \right\}$$

6.

$$\int_V C_{XX}(\underline{r}) d^3 r = \frac{1}{V} < (\Delta x)^2 > \qquad n(\underline{r}) = \sum_{j=1}^{N} < \delta(\underline{R}_j - \underline{r}) >$$

$$C_{nn} = \bar{n}^2 g(\underline{r} - \underline{r'}) \qquad < (\Delta N)^2 > = V n^2 k_B T \kappa_T$$

$$\int d^3 r [g(r) - 1] = \kappa_T k_B T - \frac{1}{\bar{n}} \qquad F(\underline{k}) = 1 + \bar{n} \underbrace{\int d^3 r e^{i\underline{k}\underline{r}} (g(\underline{r}) - 1)}_{Fourier}$$

$$< n(\underline{k}) n(-\underline{k}) > - n\delta(\underline{k}) = F(\underline{k}) \qquad \sigma(\vartheta) = \sigma_0(\vartheta) < \exp(i\underline{k}(\underline{R}_j - \underline{R}_i)) >$$

$$\sigma(\vartheta) = \sigma_0(\vartheta) F(\underline{k}) \overline{N} \qquad \bar{x}_F - \bar{x}_0 = \chi \mathfrak{I}$$

$$\chi = \left.\frac{\partial \bar{x}}{\partial \Im}\right|_{\Im=0} = \beta\left\langle (\Delta x)^2 \right\rangle_0 \qquad \chi = \beta V C_{xx}(\underline{k}=0)$$

7.

$$U(r) = 4\cdot\varepsilon\left(\left(\frac{\sigma}{r}\right)^{12} - \left(\frac{\sigma}{r}\right)^{6}\right) \qquad \frac{dp}{dT} = \frac{\Delta H}{T\Delta V}$$

$$p_c = \frac{a}{17b^2} \quad n_c = \frac{1}{36} \quad k_b T_c = \frac{8a}{27b} \quad \frac{n_c k_B T_c}{p_c} = \frac{8}{3} \quad \left(p^* + 3n^{*2}\right)\left(\frac{3}{n^*} - 1\right) = 8T^*$$

8.

$$<\sigma>= th(\beta h) \qquad M = N\mu <\sigma> \qquad \chi_T(B) = N\mu^2\beta(1 + th^2(\beta h))$$

$$H = -J\sum_{<i,j>}\sigma_j\sigma_i - h\sum_i \sigma_i \qquad Z = \sum_{\{\sigma_i\}}\exp\left(J\sum_{<ij>}\sigma_i\sigma_j + \beta h\sum_i \sigma_i\right)$$

$$H = -\sum_i \sigma_i\left(h + \frac{J}{2}\sum_{j=i+nn}\sigma_j\right) \qquad k_B T_C = \frac{J}{2}Z$$

$$\chi = \left(\frac{\partial M}{\partial B}\right)_T = N\mu^2\left(\frac{\partial <\sigma>}{\partial h}\right)_T \qquad <\sigma>= \sqrt{\frac{T_C - T}{T_C}\cdot 3} \qquad \chi_T = \frac{N\mu^2}{k_B}\frac{1}{T - T_C}$$

9.

$$f = f_0 + A(T - T_C)o^2 + Bo^4 - ho \qquad \sqrt{\frac{A}{2B}(T - T_C)} = o \qquad o = \frac{h}{2A(T - T_C)}$$

$$\frac{\chi}{n} = \frac{\beta}{h}\int C_{00}(r)d^3r = \frac{\beta}{N} < (\Delta o)^2 >$$

10.

$$\left(\frac{\partial T}{\partial S}\right)_B \left(\frac{\partial S}{\partial B}\right)_T \left(\frac{\partial B}{\partial T}\right)_S = -1$$

$$Z_N = \sum_{\substack{\{n_m\} \\ \sum_m n_m = N}} e^{-\beta\sum_m n_m\varepsilon_m} \qquad \mathcal{Z} = \prod_{m=0}^{\infty}\sum_{n=0}^{n_{\max}} e^{-\beta(\varepsilon_m - \mu)} = \prod_{m=0}^{\infty} \mathcal{Z}_m$$

$$\sum_m = g\frac{V}{h^3}\int d^3p \qquad g\frac{V}{h^3}4\pi p^2 dp = \rho(\varepsilon)d\varepsilon$$

$$\rho(\varepsilon) = g\frac{V}{h^3}4\pi(2m)^{3/2}\frac{\sqrt{E}}{2} = g\,2\pi V\left(\frac{2m}{h^2}\right)^{3/2}\sqrt{E}$$

$$g\frac{V}{h^3}\frac{2}{3}\int_0^{\infty} 4\pi p^2\, 2\bar{n}(\varepsilon)dp = \frac{2}{3}\bar{E}$$

11.

$$\phi_{klassz} \cong \pm k_B T \sum_m \overline{n}_m \approx -k_B T e^{\mu\beta} Z_1 \qquad Z_1 = g\frac{V}{h^3}(2\pi m k_B T)^{3/2}$$

$$\lambda_+ = \frac{h}{\sqrt{2\pi k_B T}} \qquad R = \left(\frac{V}{N}\right)^{1/3}$$

$$\overline{n}(\varepsilon) = e^{-\beta\varepsilon_m} e^{\beta\mu}\left(1 \mp e^{-\beta\varepsilon_m} e^{\beta\mu}\right) \qquad N \approx gV\left(\frac{2m\pi k_B T}{h^2}\right)^{3/2} e^{\beta\mu_{kl}}$$

$$e^{\beta\mu_{kl}} = \frac{1}{g}\left(\frac{\lambda_T}{R}\right)^3 \to e^{\beta\mu_{kl}} \approx e^{\beta\mu}\left(1 \mp 2^{-3/2} e^{\beta\mu}\right)$$

$$\overline{E} = 2\pi g \frac{V}{h^3}(2m)^{3/2}\frac{3}{4}\sqrt{\pi}(k_B T)^{5/2} e^{\beta\mu}(1 \mp 2^{-5/2} e^{\beta\mu}) \to \frac{\overline{E}}{\overline{N}} = sorfejtés = \frac{3}{2}k_B T(1 \pm 2^{-5/2}\frac{1}{g}\left(\frac{\lambda_T}{R}\right)^3)$$

$$p = \frac{N}{V}k_B T\left(1 \pm \frac{2^{-5/2}}{g}\left(\frac{\lambda_T}{R}\right)^3\right)$$

$$N = \sum_{|p| \in p_F} 1 = g\frac{V}{h^3}\frac{4\pi}{3}p_F^3 \qquad \mathrm{E} = g2\pi\frac{V}{h^3}(2m)^{3/2}\frac{2}{5}\varepsilon_T^{5/2}$$

$$N = g2\pi\frac{V}{h^3}(2m)^{3/2}\frac{2}{3}\varepsilon_T^{3/2} \qquad I(y) = \int_0^\infty \frac{\varepsilon^y}{e^{\beta(\varepsilon-\mu)}+1}d\varepsilon \approx \frac{\mu^{y+1}}{y+1}\left(1 + A_2(y)\left(\frac{k_B T}{\mu}\right)^2\right)$$

$$N_{T>0} = cI(y) = c\mu^{3/2}\frac{2}{3}\left(1 + A_2(y = 1/2)\left(\frac{k_B T}{\mu}\right)^2\right)$$

$$\frac{E}{N} = \mu^{3/2}\left(1 + k + \left(\frac{k_B T}{\mu}\right)^2\right) = \frac{3}{2}\varepsilon_F\left(1 + k\left(\frac{T}{T_F}\right)^2\right) \qquad C_v = \left(\frac{\partial E}{\partial T}\right)_{N,V} = Nk_B const\left(\frac{T}{T_F}\right)$$

12.

$$N(\varepsilon > 0) = N_C(T) = N_C(T_0)\left(\frac{T}{T_0}\right)^{3/2} \qquad \frac{N_0}{N} = 1 - \left(\frac{T}{T_0}\right)^{3/2}$$

$$d\upsilon \cdot n(\upsilon) = áll.(k_B T)^4 \frac{\eta^3}{e^\eta - 1}d\eta$$

$$\overline{N} = \int_0^\infty N(\varepsilon)\overline{n}(\varepsilon)d\varepsilon = áll.(k_B T)^3 V$$

$$p = \frac{1}{3}\frac{\overline{E}}{V} = \ldots = \frac{4\sigma}{3c}T^4 \qquad C = \left(\frac{\partial E}{\partial T}\right)_V = áll.VT^3$$

13.

$$\left\langle \Delta\hat{B}(t)\right\rangle = \int_{t_o\to-\infty}^{t} \varphi_{BA}(t-\tau)F(t)d\tau \qquad \left|\tilde{\psi}(t)\right\rangle = e^{\frac{i}{\hbar}\hat{H}_o t}\left|\psi(t)\right\rangle$$

$$\hat{A}(t) = \exp\left(\frac{i}{\hbar}\hat{H}_0 t\right)\hat{A}\exp\left(-\frac{i}{\hbar}\hat{H}_0 t\right) \qquad \frac{\partial}{\partial t}\left|\tilde{\psi}(t)\right\rangle = \frac{i}{\hbar}F(t)\hat{A}(t)\left|\tilde{\psi}(t)\right\rangle$$

$$\left\langle \tilde{\psi}(t)\middle|\hat{B}(t)\middle|\tilde{\psi}(t)\right\rangle \approx$$

$$\left\langle \tilde{\psi}_k\middle|\hat{B}\middle|\tilde{\psi}_k\right\rangle + \underbrace{\frac{i}{\hbar}\int_{t_o}^{t}\left\langle \tilde{\psi}_k\middle|\hat{B}(t)\hat{A}(t')\middle|\tilde{\psi}_k\right\rangle F(t')dt' - \frac{i}{\hbar}\int_{t_o}^{t}\left\langle \tilde{\psi}_k\middle|\hat{A}(t')\hat{B}(t)\middle|\tilde{\psi}_k\right\rangle F(t')dt'}_{\frac{i}{\hbar}\int_{t_o}^{t}\left\langle \tilde{\psi}_k\middle|\left[\hat{B}(t),\hat{A}(t')\right]\middle|\tilde{\psi}_k\right\rangle F(t')dt'} + \ldots$$

$$\left\langle \Delta B(t)\right\rangle = \left\langle \psi(t)\middle|\hat{B}\middle|\psi(t)\right\rangle - \underbrace{\left\langle \psi_k\middle|\hat{B}\middle|\psi_k\right\rangle}_{statfiz.átlag} = \frac{i}{\hbar}\int_{t_o\to\infty}^{t}\left\langle \tilde{\psi}_k\middle|\left[\hat{B}(t),\hat{A}(t')\right]\middle|\tilde{\psi}_k\right\rangle F(t')dt'$$

$$\varphi_{BA}(t) = \frac{i}{\hbar}\left\langle\left[\hat{B}(t),\hat{A}(0)\right]\right\rangle_o$$

$$F(t\,|\,\omega) = \lim_{\varepsilon\to+0} F_\omega e^{-i\omega t+\varepsilon t}$$

$$\left\langle \Delta B(t\,|\,\omega)\right\rangle = \lim_{\varepsilon\to+0} F_\omega e^{-i\omega t+\varepsilon t}\int_{-\infty}^{t}\varphi_{BA}(t-t')e^{i\omega(t-t')-\varepsilon(t-t')}dt'$$

$$\chi_{BA} = \lim_{\varepsilon\to+0}\int_0^\infty \varphi_{BA}(t)e^{i\omega t-\varepsilon t}dt$$

$$\left\langle \hat{B}(t)\hat{A}(0)\right\rangle_o = \sum_{mn}\frac{1}{Z}e^{-\beta E_n}B_{nm}A_{mn}e^{\frac{i}{\hbar}t(E_n-E_m)} \qquad \left\langle \hat{A}(0)\hat{B}(t)\right\rangle_o = \sum_{mn}\frac{1}{Z}e^{-\beta E_m}B_{nm}A_{mn}e^{\frac{i}{\hbar}t(E_n-E_m)}$$

$$\lim_{\varepsilon\to+0}\frac{1}{f(\omega)+i\varepsilon} = P\left(\frac{1}{f(\omega)}\right) - i\omega\delta(f(\omega))$$

$$\chi'_{BA}(\omega) = -\frac{1}{\hbar}\sum_{mn}\frac{e^{-\beta E_n}-e^{-\beta E_m}}{Z}B_{nm}A_{mn}P\left(\frac{1}{\omega+\frac{E_n-E_m}{\hbar}}\right)$$

$$\chi''_{BA}(\omega) = +\frac{\pi}{\hbar}\sum_{mn}\frac{e^{-\beta E_n}-e^{-\beta E_m}}{Z}B_{nm}A_{mn}\delta\left(\omega+\frac{E_n-E_m}{\hbar}\right)$$

$$C_{BA}(t) = \frac{1}{2}\left\langle \hat{B}(t)\hat{A}(0)+\hat{A}(0)\hat{B}(t)\right\rangle_o \qquad S_{AB} = \pi\frac{1+e^{-\beta\hbar\omega}}{Z}\sum_{mn}B_{nm}A_{mn}\delta\left(\omega+\frac{E_n-E_m}{\hbar}\right)$$

$$\underbrace{S_{BA}(\omega)}_{} = \underbrace{\hbar\cdot cth\frac{\hbar\omega}{2k_BT}}_{}\underbrace{\chi''_{BA}(\omega)}_{} \qquad \dot{E} = \frac{1}{2}F_\omega^2\omega\chi''_{AA}$$

14.

$$n(\underline{r},t)=\int n(r',t_o)\underbrace{P(\underline{r},t\,|\,r',t_o)}d^3r$$

$$P(\underline{r},t\,|\,r',t_o)=\frac{1}{(4\pi D(t-t_o))^{3/2}}e^{-(\underline{r}-\underline{r}')^2/4D(t-t_o)}$$

$$\left\langle(\underline{r}-\underline{r}')^2\right\rangle=6D(t-t_0) \qquad \dot{V}(t)=\underbrace{-\gamma V(t)}+\underbrace{\eta(t)}$$

$$V(t)=e^{-\gamma(t-t_o)}V_o+\int_{t_o}^{t}dt'e^{-\gamma(t-t')}\eta(t') \qquad \left\langle\eta(t_t)\eta(t_2)\right\rangle:=A\delta(t_1-t_2)$$

$$\left\langle V(t)V(t')\right\rangle=e^{-\gamma(t+t'-2t_o)}\left\langle V_o\right\rangle^2+AI \qquad \mathrm{I}=\int_{t_o}^{t}d\tau\int_{t_o}^{t}dt'e^{-\gamma(t-\tau)}e^{-\gamma(t'-\tau')}\delta(\tau-\tau')$$

$$I=\frac{1}{2\gamma}\left[e^{-\gamma|t-t'|}-e^{-\gamma(t+t'-2t_o)}\right] \qquad \psi(t,t')=\frac{k_BT}{M}e^{-\gamma|t-t'|}$$

$$S_1=\frac{1}{\gamma}\int_{t_o}^{t}d\tau\cdot e^{-\gamma\tau}\left[e^{\gamma\tau}-e^{\gamma t_o}\right]=\frac{1}{\gamma}(t-t_o)+\frac{1}{\gamma^2}e^{\gamma t_o}\left[e^{-\gamma t}-e^{-\gamma t_o}\right]$$

$$\left\langle(\Delta x)^2\right\rangle=2\frac{k_BT}{M\gamma}\left[(t-t_o)-\frac{1}{\gamma}\left(1-e^{-\gamma(t-t_o)}\right)\right]$$

$$M\gamma=6\pi\upsilon a$$

$$E_0=\frac{me^4}{2(4\pi\varepsilon_0)^2\hbar^2}$$

$$\phi_B=<\frac{1}{2}\sum_i\sum_j\overline{F}_{ij}^B(\overline{r}_i-\overline{r}_j)\overline{r}_i>$$

$$S=Nk\left(\frac{5}{2}+\ln\frac{V}{\sigma N}\left[\frac{4\pi mE}{3N}\right]^{3/2}\right) \qquad S'=Nk\left(\frac{3}{2}+\ln\frac{V}{\sigma}\left[\frac{4\pi mE}{3N}\right]^{3/2}\right)$$

$$T=\frac{2\varepsilon_0}{k_B}\frac{1}{\ln(N_-/N_+)} \qquad E=-N\varepsilon_0 th\frac{\varepsilon_0}{k_BT} \qquad C=-N\varepsilon_0\frac{d}{dT}th\frac{\varepsilon_0}{k_BT}$$

$$V_c=3bN \qquad p_c=\frac{a}{27b^2} \qquad RT_c=\frac{8a}{27b} \qquad \left(p^*+\frac{3}{V^{*2}}\right)\left(V^*-\frac{1}{3}\right)=8T^*$$

$$\sigma(\omega)=-i\omega(\varepsilon_r(\omega)-1)=\chi_{jp}(\omega)\int_0^\infty dt\exp(i\omega t-kt)\varphi_{jp}(t)=\varphi_{jp}\frac{\tau}{1-i\omega\tau}=\frac{e^2N\tau}{m}\frac{1}{1-i\omega\tau}$$