Relationship between particle size and normal force

János Török [1,2,*], Tamás Unger[2], Sándor Fazekas [3] & Dietrich E. Wolf[2]
[1] *Department of Chemical Information Technology, Budapest University of Technology and Economics, H-1111 Budapest, Hungary*
[2] *Department of Physics, University Duisburg-Essen, D-47048 Duisburg, Germany*
[3] *Department of Theoretical Physics, Budapest University of Technology and Economics, H-1111 Budapest, Hungary*

Static, non-cohesive, granular packings are investigated for the relationship between particle size and normal force. Our aim is to predict the average normal force acting on a given particle size from macroscopic parameters. We show that under convenient normalization a universal curve can be drawn which is insensitive to variations of friction coefficient, boundary conditions, gravity, etc. We also show that the average maximal normal force is determined solely by the geometry of the packing.

1 INTRODUCTION

If the particle size in a granular mixture has a broad distribution the forces that appear on larger particles can be orders of magnitude larger than on the small ones. This phenomenon is important, because it may lead to preferential fragmentation of the large particles in a mixture. A better understanding of this phenomenon may also help to protect a fragile component by embedding it into a suitably composed mixture. In spite of their importance, these issues have been addressed only rarely so far (1; 2; 3; 4; 5; 6).

Of course, all parameters of the sample play a role here. Taking into account all of them is impossible. Instead we would like to focus on a relation that may have a larger error bar but is independent of most parameters, like density, friction, boundary condition and particle size distribution.

This work is done by numerical simulations of static packings with distinct element methods (molecular and contact dynamics). We introduce our universal quantity that was derived from the simulation results we show its applicability and limitations.

2 DEFINITIONS

We use numerical simulations, contact and molecular dynamics to study static packings. The normal forces among particles are recorded. We define here the quantities that were found to be largely invariant from packing to packing.

The packing consists of N particles with a size distribution $P(r)$. The average particle radius is denoted by:

$$\langle r \rangle = \int_{r_{min}}^{r_{max}} rP(r)dr \qquad (1)$$

We do a convenient binning where each bin i is represented by its average radius r_i ($i = 1, 2, \dots$). The number of particles is N_i. The reduced particle radius is defined as

$$\tilde{r}_i = r_i / \langle r \rangle \qquad (2)$$

For each particle k we record the *maximal* normal force $F_{max}(k)$ acting on it. The average maximal force is thus defined as

$$\langle F_{max} \rangle = \frac{1}{N} \sum_{k=1}^{N} F_{max}(k) \qquad (3)$$

The average maximal force for bin i is

$$\langle F_{max,i} \rangle = \frac{1}{N_i} \sum_{k \in \{r_i\}}^{N} F_{max}(k) \qquad (4)$$

The reduced maximal normal force is

$$\tilde{F}_{max,i} = \langle F_{max,i} \rangle / \langle F_{max} \rangle \qquad (5)$$

3 UNIVERSAL PLOT

On Figure 1 we show the results of about 100 simulations with bidisperse and polydisperse systems with power law, Gaussian and exponential size distribution, with different friction coefficient and with and without gravity. Within an acceptable error bar the points fall on a straight line.

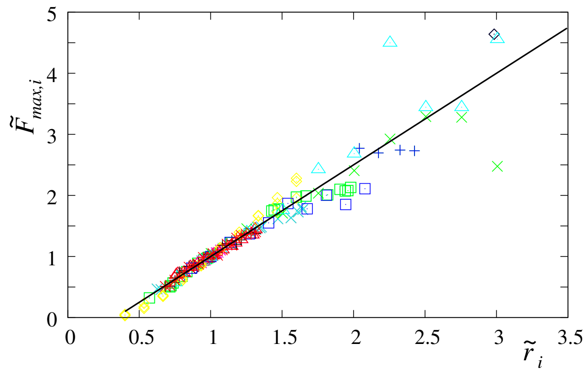

Figure 1: Reduced maximal normal force is plotted versus reduced particle radius. Symbols are results of some 100 simulations. The solid line is $y = 1.5x - 0.5$

Let us note that the error bars become quite large for large reduced radii. In these cases two problems hinder the more precise numerical simulations. First, the number of big particles must be very limited therefore the statistics are very bad, second, the simulation of systems with wide distribution is extremely time consuming.

In spite of these facts and the limitations described below it is striking that so different systems can be described by a single graph. There is no a priory constraint that requires a unique function for all particle size distributions. However, if we take this nontrivial result for granted, we can prove that this function must be linear:

$$\tilde{F}_{max,i} - 1 = a(\tilde{r}_i - 1) \qquad (6)$$

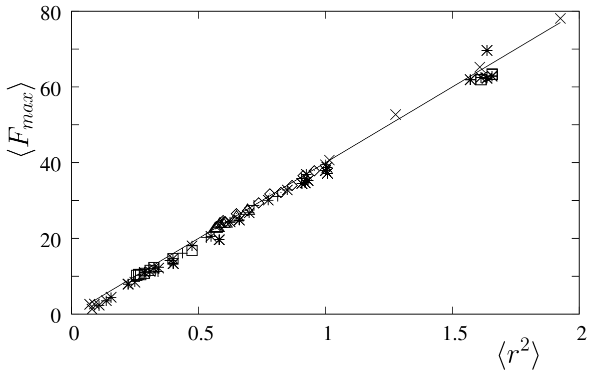

Figure 2: The average maximal normal force devided by the pressure versus the mean square radius. The points are results of more than 100 simulations. The solid line is a fit of a linear that goes through the origin.

4 MAXIMAL FORCE

The graph shown on Fig. 1 is very difficult to read. The average particle radius is generally accessible but the average maximal normal force is not. However, an important relation can be found: If the configuration of the system is fixed we found

$$\langle F_{max} \rangle \propto p \langle r^2 \rangle \tag{7}$$

where p is the external pressure (see Fig. 2). It tells us that the average maximal normal force in a system is inversely proportional to the number of particles at a crossection of the system, which is reasonable.

The above result can be used to determine how the average maximal force changes on a given particle size range for different size distributions. If the monodisperse system is accessible it gives straightforward results.

With an easy calculation that will be presented elsewhere it is possible to show that one can protect particles (reduce the average maximal normal force) with adding both smaller and bigger particles. If many smaller particles are added the reference particles will sit in the pool of small particles that transmit the load by many small contact force.

If larger particles are added at first the average maximal normal force on the reference particle is increased as the number of contacts is decreased but as more bigger particles are added the reference particles will more and more sit in the holes. In general this will remain just a theoretical case as described in the next section. But with carefully crafted distributions this option can also be used.

5 LIMITATIONS

We note that not all of our simulation results are plotted on Fig. 1. We omitted cases where a certain type of particles did not participate in the force bearing network. It is easy to see that if one adds to the system small force-free particles the resulting points will not be on the same line.

It is possible to calculate this effect and compare it to simulations. Let us consider a monodisperse system ($r_{big} = 1$) to which we add small particles (r_{small}) which may fit into the holes of the big ones. Of course most of the small particles will be force-free thus $\tilde{F}_{max,small} = 0$. The average maximal normal force on the big particles reads as

$$\tilde{F}_{max,big} = \frac{1 - r_{small}}{1/\tilde{r}_{big} - r_{small}}. \tag{8}$$

On Fig. 3 simulations of a bidisperse system is shown where the ratio of the radii is

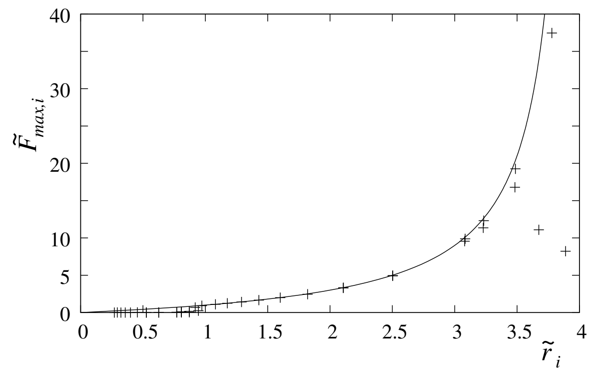

Figure 3: Reduced maximal normal force is plotted versus reduced particle radius. Results of 19 bidisperse system are plotted with particle diameters 1 and 0.25. The ratio of the big particles for the rightmost two points is less the 1%.

4. If the number of the big particles amount to more than 1% then the small particles are basically force-free and the data for the large particles fits quite well Eq. (8).

Another important limitation is that the line that seems to fit the simulation data on Fig. 1 has negative values for small $\tilde{r}_i$. One may think that it is easy to construct packings with very low $\tilde{r}_{i,min}$. But if we require that even the smallest particles should carry forces and are not allowed to sit in holes then one can easily realize that it puts up a limit. From Apollonian packings (7; 8) one gets a minimal $\tilde{r}_{i,min} = 0.59$. Of course these packings are ordered and if this is not required one can a better estimate for $\tilde{r}_{i,min}$ which is closer to our empirical fit of 1/3.

6 CONCLUSION

In this paper we showed that a universal curve can be drawn that describes quantitatively how particles with different size carry load in a static packing. We showed that this universal curve is not sensitive to many material parameters. It has an important limitation when certain particles become force-free. We showed that in this case another function will be obtained. We showed that the average maximal normal force scales linearly with the average particle radius square and that this makes the universal curve applicable in practice.

We mentioned that in certain circumstances reference particles can be protected by either smaller or bigger particles however the latter is more difficult to achieve.

Part of this work was supported by OTKA F047259, by the BMBF through grant HUN 02/011, and by Federal Mogul GmbH.

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