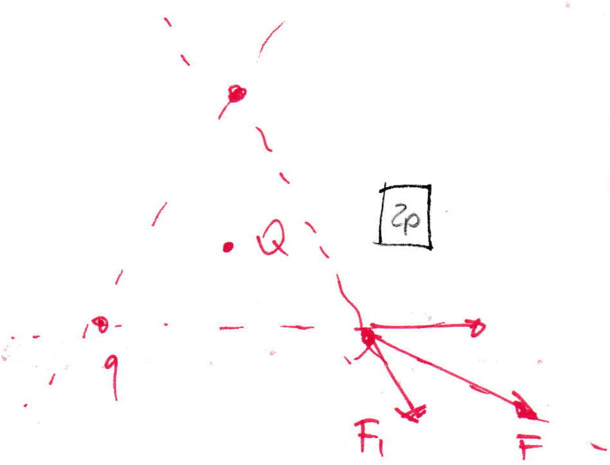
①



2p

$F = 2\cdot\cos 30°\cdot F_1 = \sqrt{3}\cdot F_1 = \sqrt{3}\,k\cdot\frac{q^2}{a^2}$ 2p

$F_1 = k\cdot\frac{q^2}{a^2}$ 2p

Középre: 1p

$F_2 = k\cdot\frac{qQ}{\left(\frac{\sqrt{3}}{2}\cdot\frac{2}{3}\cdot a\right)^2} = 3k\,\frac{qQ}{a^2}$ 3p

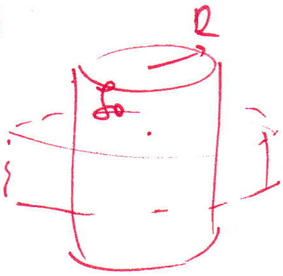
3p

$\sqrt{3}\,k\,\frac{q^2}{a^2} = 3k\,\frac{qQ}{a^2}$

$\boxed{\frac{1}{\sqrt{3}}q = Q}$ 2p

és ~~q~~ q Q ellentétes előjelű. 1p

②



2p D: $\oint_{\partial V} \underline{D}\, d^2\underline{f} = \iiint_V \rho(\underline{r})\, d^3\underline{r}$

$\underline{E}/\underline{D}$ hengerszimmetrikus

$\underline{E} \parallel \underline{e}_r$

$\underline{E}(\underline{r}) = E(r)\cdot\underline{e}_r$ 2p

Henger: $(r<R)$

$\emptyset + \emptyset + 2r\pi\cdot\ell\cdot D(r) = r^2\pi\cdot\ell\cdot\rho_0$

$D(r) = \frac{1}{2}\rho_0\cdot r$ 4p

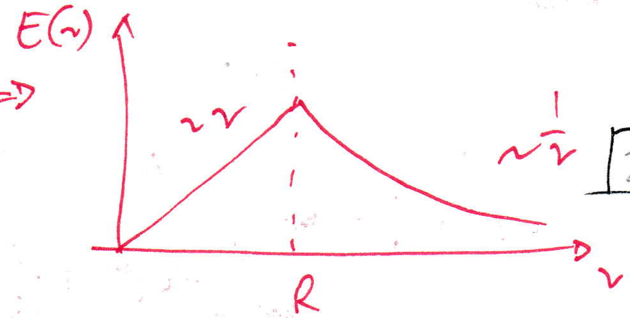
$(r>R)$

$2r\pi\ell\cdot D(r) = R^2\pi\ell\rho_0$

$D(r) = \frac{R^2}{r}\cdot\frac{\rho_0}{2}$ 4p

$\Rightarrow E = \begin{cases} \frac{1}{2\varepsilon_0}\rho_0 r \\ \frac{1}{2\varepsilon_0} R^2\rho_0\cdot\frac{1}{r} \end{cases}$

$\Rightarrow$



2p

$F = E(d)\cdot q = \frac{1}{2\varepsilon_0}\cdot(0{,}1\,m)^2\cdot 10\frac{C}{m^3}\cdot\frac{1}{\cancel{0{,}2\,m}}\ \cancel{0{,}2C}$

$= \frac{1}{2\varepsilon_0}\cdot 0{,}1\,\frac{C^2}{m^2} = \dots$ 2p

③ $u(xyz) = \alpha z^2 \cos\left(2\pi \frac{x}{x_0}\right) + \beta z e^{-\frac{y}{y_0}}$

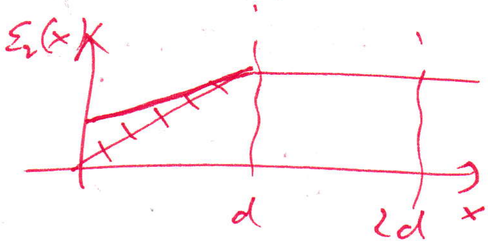
$$\underline{E}(xyz) = -\underline{\nabla} u(xyz) = \boxed{3p}$$

$$= -\left(-\alpha z^2 \frac{2\pi}{x_0} \cdot \sin\left(2\pi \frac{x}{x_0}\right);\ -\frac{\beta z}{y_0} e^{-\frac{y}{y_0}};\ 2\alpha z \cos\left(2\pi \frac{x}{x_0}\right) + \beta e^{-\frac{y}{y_0}} \right)$$

$\boxed{3p}$ $\boxed{3p}$ $\boxed{3p}$

Elöjelhiba (−3p)

④ $\varepsilon_r(x) = \begin{cases} \frac{\varepsilon_{2r} - \varepsilon_{1r}}{d} \cdot x + \varepsilon_{1r} \\ \varepsilon_{2r} \end{cases}$ $\boxed{3p}$



$$D = \sigma = \frac{Q}{A} \quad \boxed{2p}$$

$$E = E(x) = \frac{D}{\varepsilon_0 \varepsilon_r(x)} = \begin{cases} \dfrac{Q}{A\varepsilon_0 \left(\frac{\varepsilon_{2r} - \varepsilon_{1r}}{d} x + \varepsilon_{1r}\right)} \\ \dfrac{Q}{A\varepsilon_0 \varepsilon_{2r}} \end{cases} \quad \boxed{2p}$$

$$\boxed{2p} \quad U = -\int_0^{2d} E(x)\,dx = -\int_0^{d} E(x)\,dx - \int_d^{2d} E(x)\,dx =$$

$$= -\int_0^d \frac{Q}{A\varepsilon_0} \frac{1}{\frac{\varepsilon_{2r} - \varepsilon_{1r}}{d} x + \varepsilon_{2r}} \cdot dx - \int_d^{2d} \frac{Q}{A\varepsilon_0 \varepsilon_{2r}} dx =$$

$$= -\frac{Q}{A\varepsilon_0} \cdot \left[\frac{\ln\left(\frac{\varepsilon_{2r} - \varepsilon_{1r}}{d} x + \varepsilon_{1r}\right)}{\frac{\varepsilon_{2r} - \varepsilon_{1r}}{d}} \right]_0^d \boxed{3p} - \frac{Q}{A\varepsilon_0 \varepsilon_{2r}} \cdot d \quad \boxed{2p}$$

$$= -\frac{Q}{A\varepsilon_0} \left[\frac{d}{\varepsilon_{2r} - \varepsilon_{1r}} \cdot \ln\left(\frac{\varepsilon_{2r}}{\varepsilon_{1r}}\right) + \frac{d}{\varepsilon_{2r}} \right] \Rightarrow \boxed{C = \frac{Q}{U} = \varepsilon_0 \cdot \frac{A}{\frac{d}{\varepsilon_{2r} - \varepsilon_{1r}} \cdot \ln\left(\frac{\varepsilon_{2r}}{\varepsilon_{1r}}\right) + \frac{d}{\varepsilon_{2r}}}} \quad \boxed{2p}$$

$\boxed{5}$

$$D(r) = \begin{cases} \emptyset & r < R_1 \\ \dfrac{Q}{4\pi} \cdot \dfrac{1}{r^2} & R_1 < r < R_3 \\ \emptyset & R_3 < r \end{cases} \quad \boxed{6} \quad \Rightarrow E(r) = \begin{cases} \emptyset & r < R_1 \\ \dfrac{Q}{4\pi\varepsilon_0\varepsilon_1} \cdot \dfrac{1}{r^2} & R_1 < r < R_2 \\ \dfrac{Q}{4\pi\varepsilon_0\varepsilon_2} \cdot \dfrac{1}{r^2} & R_2 < r < R_3 \\ \emptyset & R_3 < r \end{cases} \quad \boxed{4}$$

$$\underline{D}(r) \sim \underline{e}_r \quad \boxed{2p}$$

$$\hookrightarrow U = -\int_{R_1}^{R_3} E(r)\,dr = -\int_{R_1}^{R_2} \frac{Q}{4\pi\varepsilon_0\varepsilon_1}\frac{1}{r^2}\,dr - \int_{R_2}^{R_3} \frac{Q}{4\pi\varepsilon_0\varepsilon_2}\frac{1}{r^2}\,dr =$$

$\boxed{2p}$ $\boxed{3p}$ $\boxed{3p}$

$$= -\frac{Q}{4\pi\varepsilon_0}\left(\frac{1}{\varepsilon_1}\left[-\frac{1}{r}\right]_{R_1}^{R_2} + \frac{1}{\varepsilon_2}\left[-\frac{1}{r}\right]_{R_2}^{R_3}\right) =$$

$$= +\frac{Q}{4\pi\varepsilon_0}\left(\frac{1}{\varepsilon_1}\left(\frac{1}{R_2} - \frac{1}{R_1}\right) + \frac{1}{\varepsilon_2}\left(\frac{1}{R_3} - \frac{1}{R_2}\right)\right)$$