STATISTICAL ANALYSIS OF THE STOCK INDEX OF THE BUDAPEST STOCK EXCHANGE

J. ROTYIS

FOCUS Investment Rating Company, H-1053 Budapest, Szép u. 2 X/1010, Hungary

AND

G. VATTAY

Eötvös University, Department of the Physics of Complex Systems, H-1117 Budapest, Pázmány P. sétány 1/A, Hungary

1. Historic Notes

The Budapest Stock Exchange [2] (BSE) opened in 1864 and had its best period around the turn of the century, when it became the fourth largest stock exchange in Europe. It was closed in 1919 for a few months, again in 1945 due to the siege of Budapest, and finally in 1948 it was permanently closed as a consequence of the communist takeover. Then in 1982 new legislation permitted issuing bonds once again. The semi-official trading of bonds was restarted in 1982 and it become possible to establish share companies. The Exchange reopened on the 19th of June, 1990 and had its first trading day on the 21st of June.

The base day of the index, the 'BUX', is January 2, 1991 when it was set to 1000 points. It is calculated through a weighted average of the prices of main stocks and represents the total capitalization of the market. The BUX index reached its minimum in May 1993 at 718 points, caused by the escalation of the war in ex-Yugoslavia. The index, after some ups and downs reached the value 1500 by the end of 1995. Since then the index has gone steadily up to about 8000 in August 1997, producing the highest (inflation adjusted) stock exchange growth in the world in 1996. In this paper we analyze the index and some of the main stocks for the period January-June 1997, a period of steady growth.

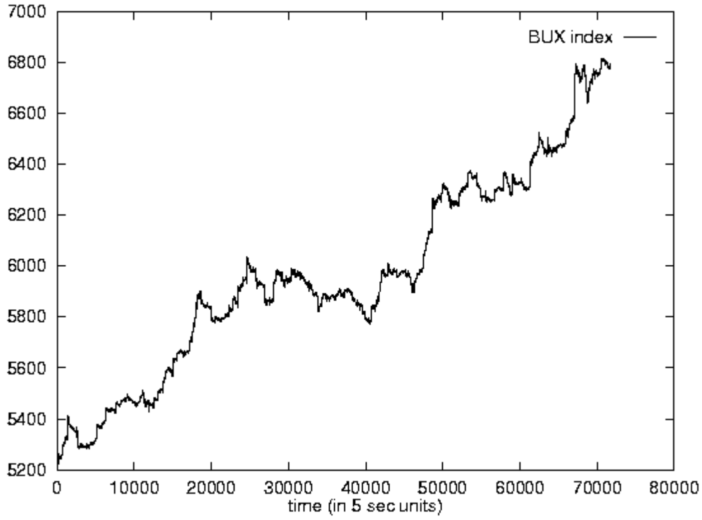

Figure 1. Real time BUX index for the period 1 of April - 30 of June 1997. We joined the daily records and produced a single data sequence. The time is measured in 5 seconds units.

2. The Data

In the present analysis we use the data provided by the BSE. Securities trading on the BSE is entirely electronic. Dealers submit their ask and bid prices and the amounts, and the computer system matches the orders between partners. The computer gives priority to higher bids and lower asks and under equal conditions time of orders also counts. The time, the amount and the price is then recorded. We have such records for the five most traded stocks of the BSE for the entire period 6 of January - 30 of June 1997 for each trading day.

Before 1 of April 1997 the index was calculated at the end of each trading day based on the market closing prices. Since then the index has been calculated continuously. On each trading day the index calculation begins 15 minutes after the opening of the market. After that it is calculated every 5 seconds. Very often no deal is made within 5 seconds and the index remains unchanged. We also have the BUX index record for the period 1 of April - 30 of June 1997. In Fig. 1 we show the evolution of the real time BUX index for this period.

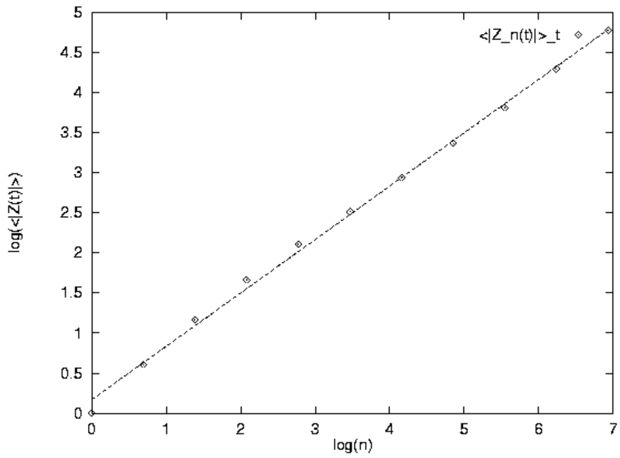

Figure 2. Scaling of the absolute value of the return. *Natural logarithms* of the quantities are plotted. The scaling $\sim n^{0.666}$ is observed for almost three decades.

3. Scaling and Distribution of Returns

Following Refs.[3, 5] we study the scaling properties of returns of the BUX index. The return over n time steps is defined as

$$Z_n(t) = X(t + n\Delta t) - X(t), \tag{1}$$

where $X(t)$ is the BUX index and $\Delta t = 5sec$ is the sampling time. We have found that the moments of the distribution of Z_n show scaling behavior as a function of n

$$\langle |Z_n(t)|^q \rangle_t \sim n^{\xi_q}, \tag{2}$$

where ξ_q is the self-affinity exponent. In Fig. 2 we show the $q = 1$ moment for $n = 1, ..., 1024$. We can observe scaling for three decades (on decimal scale). We have found that this remarkable agreement with the scaling assumption (2) holds for q values larger than 0.2 . Traditional stock market theory [4] (TSMT) predicts Brownian motion for the stock prices which leads to a uniform exponent $\xi_q = q/2$. In Fig. 3 we show the measured exponents ξ_q as a function of q. The function ξ_q is strongly non-linear and differs strongly from the behavior of the S&P 500 index in Ref.[5] and DM/US$ exchange rate in Ref.[3].

We can see that the exponent for $q = 1$ is close to 2/3, deviating significantly from TSMT, while for $q = 2$ the exponent is about 1, predicted by TSMT. This is very puzzling, since traditionally the volatility is measured

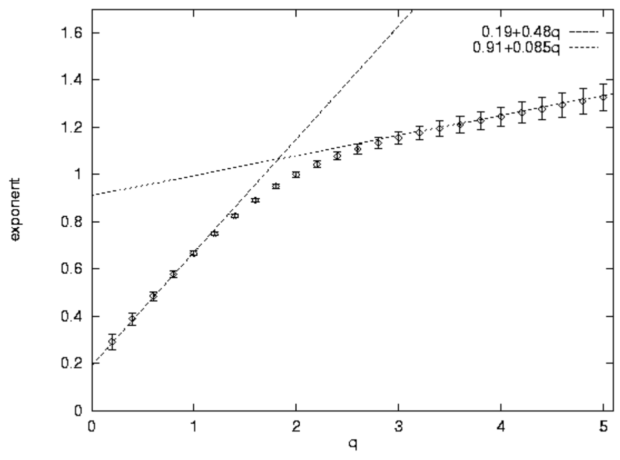

Figure 3. Measured values of ξ_q for the BUX index. The errorbars represent the error of the fit of the power law.

via the relation

$$< |X(t+n\Delta t) - X(t)|^2 >\sim \sigma^2 n, \tag{3}$$

which seems to hold for the BUX index, despite the strongly non-Brownian behavior of other cummulants.

It is even more surprising that the growth rate seems to follow TSMT. In Fig. 4 we show the average behavior of $\log X(t+n\Delta t) - \log X(t)$. We have found that

$$\langle \log(X(t+n\Delta t)/X(t)) \rangle \approx \mu n, \tag{4}$$

where $\mu \approx 0.074 = e^{-2.60}$ so that the expected behavior of the index is

$$X(t+n\Delta t) \approx X(t)e^{\mu n}. \tag{5}$$

Next, we studied the distribution $P_n(Z)$ of the returns $Z_n(t)$ shown in Fig. 5. These curves are similar to those observed for the S&P index in Ref.[5]. The center of the distribution shows similar scaling $P_n(0) \sim n^{-1/\alpha}$, where the exponent is $1/\alpha \approx 0.666$, $\alpha \approx 1.50$ in accord with the value $\xi_1 \approx 0.66$ and close to the S&P value $\alpha = 1.40$ of Ref.[5] and coincides with the exponent found by Bouchaud et al. in Ref.[6]. The distribution, however, cannot be fitted with a Lévy type distribution since the rescaled curves $P_n(Z/n^{0.66})n^{0.66}$ do not collapse onto a single curve. This fact is in accord with the strong non-linearity observed in ξ_q. We can say qualitatively that the distributions are similar to Lévy distributions with parameter $\alpha = 1.5$

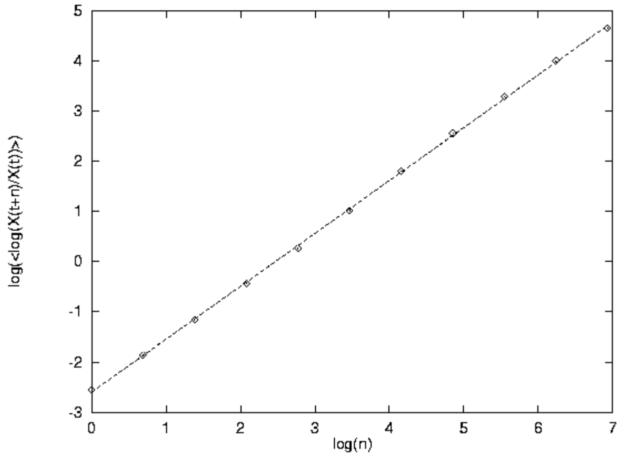

Figure 4. Average value of $<\log(X(t+n\Delta t)/X(t))>$ for $n = 1, 2, ..., 1024$ and $\Delta t = 5sec$. The best fit line represents $X(t + n\Delta t) = e^{0.074n}X(t)$.

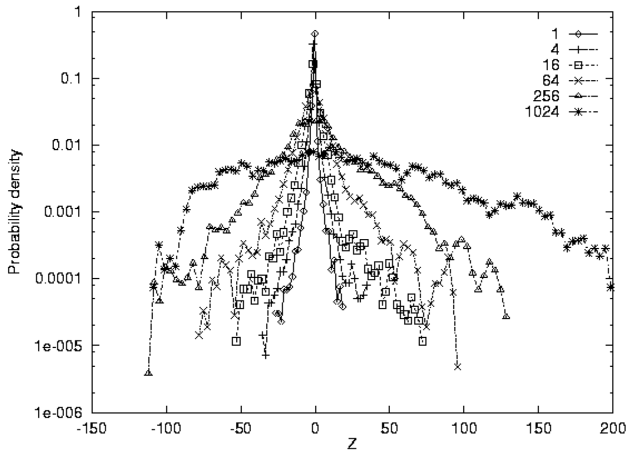

Figure 5. The probability density $P_n(Z)$ on log-linear plot for $n = 1, 4, 16, 256, 1024$.

up to times $\approx 300sec$ and Gaussian (log-normal) distribution sets in around $\approx 5000sec$.

Subsequently we also studied the scaling behavior of the main stocks on BSE which lead to similar exponents and conclusions.

4. Correlations and Fourier Analysis

Temporal correlations or their absence play an important role in the predictability of the market. Here we study the behavior of the correlation functions

$$C_n(m) = \langle Z_n(t + m\Delta t) Z_n(t) \rangle_t. \tag{6}$$

According to TSMT short time increments of stock prices are uncorrelated so the normalized autocorrelation function of the single increments is

$$C_1(m)/C_1(0) = \delta_{0,m}. \tag{7}$$

We tested this hypothesis for the BUX index and found perfect agreement. The absence of short time correlations can be the result of the random match-making process between dealers mentioned in the previous section. Then the increments $X(t+n\Delta t) - X(t)$ can be considered as a sum of $n-1$ uncorrelated steps. The shifted series $X(t + (m + n)\Delta t) - X(t + m\Delta t)$ is composed of $n - 1$ uncorrelated increments, however $n - 1 - m$ of these increments coincide with those in $X(t + n\Delta t) - X(t)$. As a consequence, the TSMT prediction for $C_n(m)$ is

$$\begin{aligned} C_n(m)/C_n(0) &= 1 - m/(n-1) \quad \text{for } m < n, \\ &= 0 \quad \text{otherwise}. \end{aligned} \tag{8}$$

In Fig. 6 we show the absolute value of the autocorrelation functions for the BUX index for $n = 100$ and $n = 1000$. We have found that correlation functions up to $n \approx 100$ behave in accord with (8) while above that correlations are higher than those predicted by TSMT. For example, the $n = 1000$ ($5000sec$) data can be well fitted with $C_{1000}(m)/C_{1000}(0) = 1 - n/1200$. This increased level of correlations can be observed by looking at the power spectrum

$$S_n(f) = \left| \int dt e^{i2\pi ft} Z_n(t) \right|. \tag{9}$$

For $n < 100$ the power spectrum is constant, indicating that the series $Z_n(t)$ is white noise. For $n \approx 1000$ the power spectrum shows scaling behavior (see Fig. 7) and it is very close to

$$S(f) \sim 1/f, \tag{10}$$

indicating the presence of noise related to one-dimensional Brownian motion.

5. Conclusions

As we demonstrated, the scaling properties of the BUX index are similar to those observed in other parts of the world. Bouchaud argued [6] that

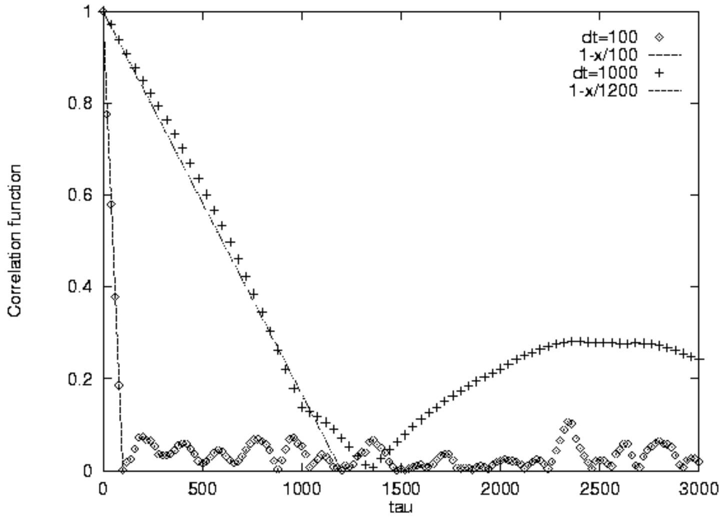

Figure 6. Autocorrelation functions $|C_{100}(\tau)/C_{100}(0)|$ and $|C_{1000}(\tau)/C_{1000}(0)|$. Notice that fluctuations of the $n = 1000$ correlation function are about $(10)^{1/2}$ times larger than those for $n = 100$ because of the higher redundancy of the series $Z_{1000}(t)$. The fluctuating parts are insignificant in both cases.

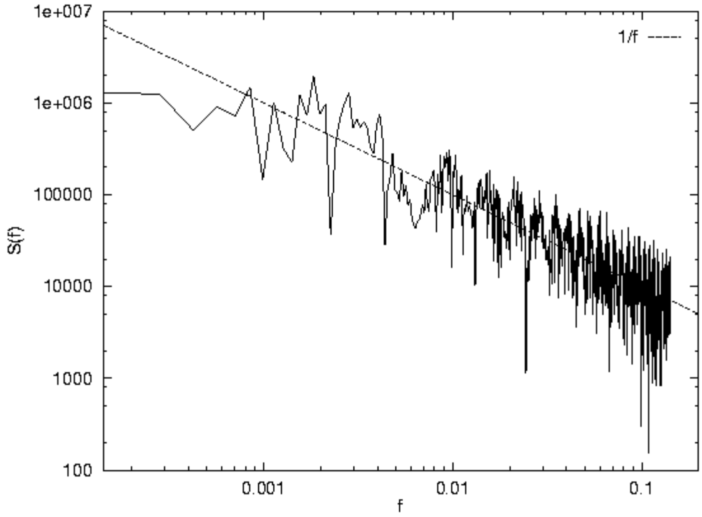

Figure 7. Power spectrum $S_{1000}(\omega)$ of $Z_{1000}(t)$. The solid line indicates $1/f$.

short time correlations are in accord with the efficient market hypothesis, provided that transaction costs make it impossible to use them for arbitrage. The observed complete absence of short time correlations on the BSE, present on other stock markets, may be due to the fact that members of the BSE were not required to pay transaction fees in the investigated period. The reason for long time (5000 sec) correlations is less clear and a more extended study is needed to find its origin.

The authors thank Ildikó Fecser, Tamás Laskai and Zsolt Kendi, the BSE team, for their support. G. V. thanks I. Csabai, B. Janecskó, I. M. Jánosi and I. Kondor for discussions and the Hungarian Science Foundation OTKA (F019266/F17166/T17493) and the Hungarian Ministry of Culture and Education for financial support.

References

1. CEO of the BSE during the period of this study.
2. This section is based on: J. Rotyis, *Handbook for Stock Exchange Investors* (in Hungarian), CONEX, Budapest (1997)
3. S. Ghashghaie, W. Breymann, J. Peinke, P. Talkner and Y. Dodge, Nature **381**, 767 (1996)
4. L. J. B. Bachelier, *Théorie de la Speculation* (Gauthier-Villars, Paris, 1900); F. Black and M. Scholes, Journal of Political Economy **3**, 637 (1973)
5. R. N. Mantegna and H. E. Stanley, Nature **376**, 46 (1995)
6. J-P. Bouchaud and M. Potters, *Théorie des Risques Financiers* (Aléa Saclay, Paris, 1997) and references therein.