DETRENDED FLUCTUATION ANALYSIS OF THE FOREIGN EXCHANGE MARKET

N.VANDEWALLE AND M.AUSLOOS
SUPRAS, Institut de Physique B5, Université de Liège, B-4000 Liège, Belgium

AND

PH.BOVEROUX
Théorie monétaire et finances B31, Université de Liège, B-4000 Liège, Belgium

1. Introduction

Scale invariance seems to be widespread in natural systems [1]. Numerous examples can be found in the literature: earthquakes, sandpiles, river networks, clouds, mountains, etc.. Such an abundance of quasifractal objects in nature is still actually a puzzling problem [1, 2, 3].

In particular, problems in economics and finance have recently started to attract the interest of statistical physicists. Fundamental problems are whether long-range power-law correlations exist in economic systems and the explanation of economic cycles. Indeed, traditional methods (like spectral methods) have corroborated that there is evidence that the Brownian motion idea is only approximately right [4, 5, 6].

Long-range power-law correlations have been discovered in economic systems and particularily in company growth [7] and in financial fluctuations [8]. Different approaches have been envisaged to measure the correlations and to analyze them. Through the so-called Lévy statistics, Stanley et al. [8] have shown the existence of long-range power-law correlations in the Standard and Poor (S&P500) index. A method based on wavelet analysis has also shown the emergence of hidden structures in the S&P500 index [9]. We have recently performed a Detrended Fluctuation Analysis (DFA) of the USD/DEM ratio [10] and we have demonstrated the existence of successive sequences of economic activity having different statistical behaviors.

In the present report, we perform a DFA of various foreign exchange rates as for example the JPY/USD and the GBP/DEM rates. This work is also an attempt to classify the behaviour of exchange rates and to provide some interpretations of apparently random fluctuations in financial data. The next section will describe the DFA technique. Information about the data will be given. Numerical results for long-range and local correlations will be given in Secs. 4 and 5, respectively. The robustness of the DFA technique will be underlined in Sec. 6 through the analysis of artificial data, and the comparison of results following different detrending functions. Additional remarks on the DFA method will be given in Sec. 7. A conclusion will be drawn in Sec. 8.

2. The DFA Analysis

The Detrended Fluctuation Analysis (DFA) technique was introduced a few years ago in order to investigate long-range power-law correlations along DNA sequences [11, 12]. The DFA method consists of dividing the whole data sequence $y(n)$ of length N into N/t nonoverlapping boxes, each containing t points. The *local trend*

$$z(n) = an + b \tag{1}$$

in each box is defined to be the ordinate of a linear least-square fit of the data points in that box. One should remark that a trend $z(n)$ different from a first-degree polynomial can also be used, such as the cubic trend [13] or an asymmetric Λ-function [14]. Cubic detrending will be discussed in Sec. 6. The use of other detrending functions may improve the accuracy of the DFA technique but this remains outside the scope of the present paper, although we suggest that some basic improvements over the method are necessary (see Sec. 7).

The detrended fluctuation function $F(t)$ is then calculated from

$$F^2(t) = \frac{1}{t} \sum_{n=(k-1)t+1}^{kt} |y(n) - z(n)|^2, \qquad k = 1, 2, \cdots, N/t \tag{2}$$

Averaging $F(t)$ over the N/t intervals gives a function depending on the box size t. The above calculation is thus repeated for different box sizes t. If the $y(n)$ data are randomly uncorrelated variables or short range correlated variables, the behavior is expected to be a power law

$$\langle F \rangle \sim t^{\alpha} \tag{3}$$

with an exponent 1/2 [11] just as if the excursion were governed by a mere random walk. An exponent $\alpha \neq 1/2$ in a certain range of t values implies the existence of long-range correlations in that time interval. Then, the signal $y(n)$ can be well approximated by the fractional Brownian motion law [15, 16]. Mathematically, the correlation of a future increment $y(n) - y(0)$ with a past increment $y(0) - y(-n)$ is then given by

$$C(t) = \frac{\langle (y(0) - y(-n))(y(n) - y(0)) \rangle}{\langle (y(n) - y(0))^2 \rangle} = 2^{2\alpha - 1} - 1 \qquad (4)$$

where the correlations are normalized by the variance of $y(n)$ [15].

The cases $\alpha > 1/2$ and $\alpha < 1/2$ should be physically distinguished [15]. For $\alpha > 1/2$, there is *persistence*, i.e. $C > 0$. In this case, if in the immediate past the signal has a positive increment, then on average an increase of the signal in the immediate future is expected. An exponent $\alpha < 1/2$ means *antipersistence*, i.e. $C < 0$. In this case, an increasing value in the immediate past implies a decreasing signal in the immediate future, while a decreasing signal in the immediate past makes an increasing signal in the future probable. In so doing, data records with $\alpha < 1/2$ appear very *noisy* (rough). They have a local noise of the same order of magnitude as the total excursion of the record [15]. The $\alpha = 0$ situation corresponds to the so-called *white noise* which is obviously antipersistent. Finally, one should note that α is nothing but H, the so-called Hurst exponent for fractional Brownian motion [15].

It can be useful to recall [17] that the power spectrum of such random signals is characterized by a power law with an exponent $\beta = 2\alpha - 1$ as investigated by Liu et al. in this volume.

3. The Collection of Data

We have considered the daily evolution of several currency exchange rates from January 1980 till December 1996 only including all open banking days. This represents about $N = 4400$ data points. The week-ends and holidays are not considered even though political or social events can occur during week-ends. The data were collected in Brussels at 02:30 p.m. local time [18], i.e. near the closing time of the foreign exchange market in London.

4. Long-range Power-law Correlations

The evolution of the JPY/USD exchange rate from 1980 to 1996 is drawn in Fig.1. In Fig.2, a log-log plot of the function $\langle F(t) \rangle$ is shown for the whole of the data set in Fig.1. This function is very close to a power law with an exponent $\alpha = 0.55 \pm 0.01$ holding over two decades in time, i.e. from about one week to two years. This finding clearly supports the existence of long-range power-law correlations in the foreign exchange market as discussed

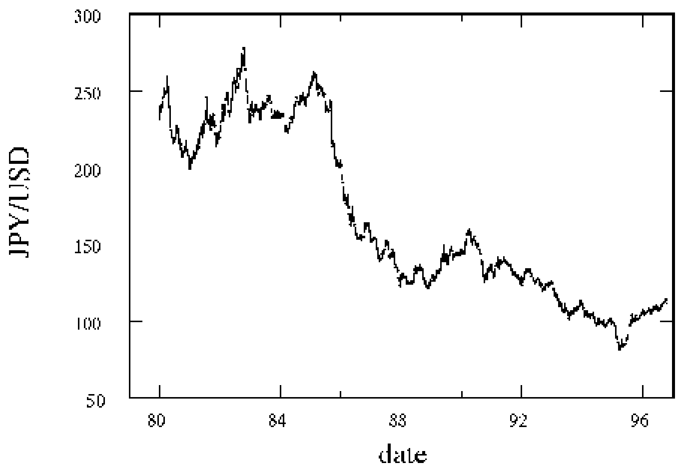

Figure 1. The daily evolution of the JPY/USD exchange rate from January 1980 to December 1996.

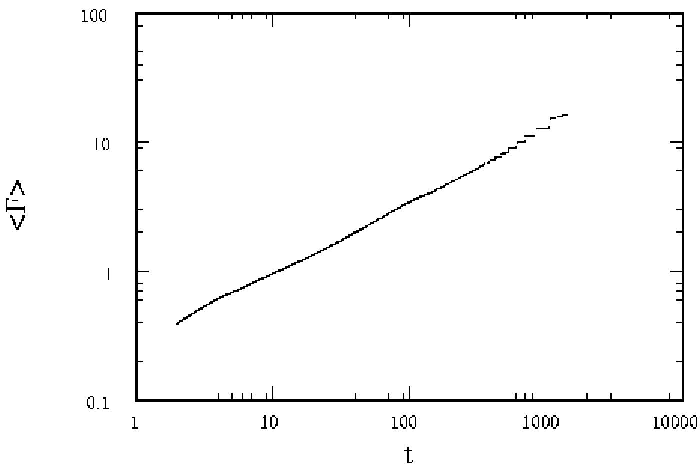

Figure 2. The fluctuations $\langle F \rangle$ as a function of the box size t for the evolution of the JPY/USD exchange rate from January 1980 to December 1996, i.e. the data of Fig. 1.

in [10]. These power laws are a signature of the propagation of *information* across the economic system during very long times (up to two years in this particular case). This is quite similar to the physical phenomenon of *anomalous diffusion* [19, 20]. Liu et al. [21] have reported similar results, i.e. long range correlations, for the intraday fluctuations of the New York Stock Exchange.

Table 1. presents the results of the DFA applied to various currency exchange rates. It is observed that a wide variety of behaviors, i.e. different

α values, can be found on the foreign exchange market. Exponent values and the range over which the power law (3) holds vary drastically from one currency exchange rate to another. Nevertheless it appears that the currency exchange rates can be put into three different categories.

TABLE 1. Values of α for various currency exchange rates obtained with the help of DFA. The range of power-law validity (Eq.(3)) is also given.

exchange rate	α	range (weeks)
USD/DEM	0.55 ± 0.02	$1 \rightarrow 50$
JPY/USD	0.55 ± 0.02	$1 \rightarrow 101$
GBP/DEM	0.55 ± 0.02	$1 \rightarrow 62$
USD/CAD	0.50 ± 0.02	$1 \rightarrow 32$
NLG/BEF	0.26 ± 0.03	$1 \rightarrow 10$
DEM/BEF	0.23 ± 0.03	$1 \rightarrow 8$
DKK/BEF	0.31 ± 0.03	$1 \rightarrow 8$
FRF/BEF	0.37 ± 0.03	$1 \rightarrow 46$
PLZ/BEF	0.33 ± 0.03	$1 \rightarrow 20$

First, there are rates which exhibit an exponent α larger than 1/2 (persistent behavior). It should be noted that these currency exchange rates involve leading currencies (USD, JPY, GBP, FRF and DEM). In general, the value of α is close to 0.55 in such cases. For all leading currencies, Eq.(3) holds over two decades, i.e. from one week to one year.

The second category concerns the rates exhibiting strict randomness ($\alpha = 1/2$) within error bars. This is the case for example of the USD/CAD rate.

The third category represents the currency echange rates with antipersistent behaviour ($\alpha < 1/2$) as e.g. DEM/BEF. These currencies most often concern exchange rates between european currencies which are submitted to strict monetary rules and to strict regulatory corrections by central banks due to international multilateral conventions. Thus, it is not surprising to obtain an antipersistent behavior with little excursion of the exchange rate in these cases.

It should be pointed out that in general the range over which the antipersistency signature, i.e. the power law, is valid occurs over a limited time span in the third category. In fact, there is a crossover around $t^* \approx 10$ weeks. For longer time scales ($t >> t^*$), the economic signal $y(t)$ becomes again persistent or random. As an illustration of this crossover to a higher exponent value of around $t^* \approx 10$ weeks, Fig.3 exhibits the fluctuation

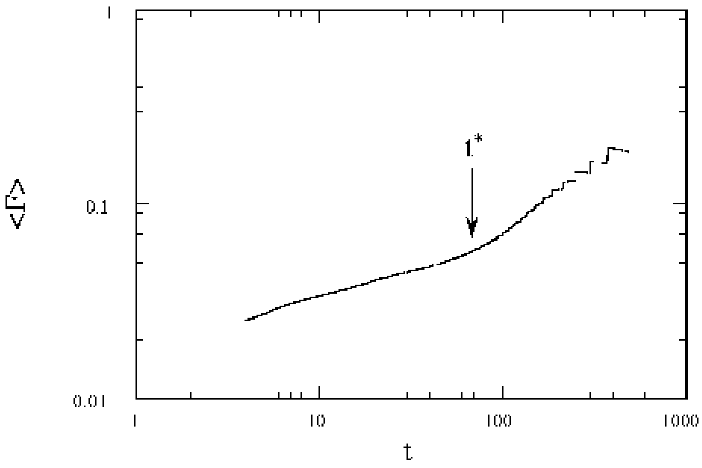

Figure 3. The fluctuations $\langle F \rangle$ as a function of t for the daily evolution of the NLG/BEF ratio during the period from January 1980 till December 1996. The arrow indicates the position of the crossover t^* from antipersistent to persistent behavior.

function $\langle F \rangle$ for the NLG/BEF ratio. This crossover is similar to what was observed by Liu et al. for intraday correlations on the New York stock exchange [21]. The existence of a crossover t^* also supports the idea that there are multiple information levels, as there are for turbulence [22, 23]. As a consequence, a hierarchy of exponents are needed instead of a single exponent α, but this is outside the scope of the present contribution.

5. Probing the Local Correlations

In the above, the DFA has been demonstrated to probe the existence of long-range power-law correlations in currency exchange rates. It is also of interest to know whether these correlations are stable along the data.

In order to probe the local nature of the correlations, we first construct a so-called observation box (a probe) of "length" T placed at the beginning of the data, and we calculate α for the data contained in that box. Then, we move this box by a few points (e.g. 4 weeks) toward the right along the financial sequence and again calculate α. Iterating this procedure for the 1980-1996 period, we obtain a "local measurement" of the degree of "long-range correlations" over T. With this method, it is crucial to choose the best box size T. It seems natural to choose T of the same order of magnitude as the maximum range t over which the power law Eq.(3) is valid.

The evolution of the GBP/DEM ratio for the 1980-96 period is illustrated in Fig. 4a. For this ratio, a global exponent $\alpha = 0.55$ has been found

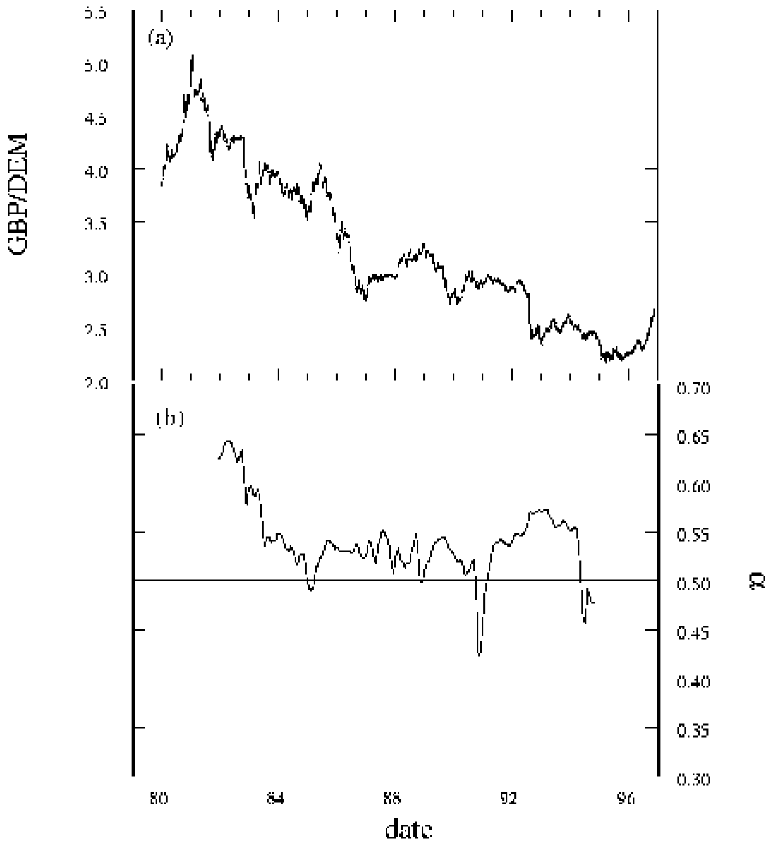

Figure 4. (a) The daily evolution of the GBP/DEM exchange rate from January 1980 to December 1996. (b) The local exponent α for the evolution of the GBP/DEM exchange rate. The upward (downward) arrows indicate increases (decreases) of the Bundesbank discount rate (given in Table 2.).

(see Table 1.). In order to probe the local values of α, we have used a window of size $T = 2$ years, i.e. roughly twice the range over which Eq.(3) is valid. Fig. 4b presents the results of this mobile Detrended Fluctuation Analysis. The exponent α is mostly above 1/2. Averaging all local values of α, one can find the global value $\alpha = 0.55$. The horizontal dashed line in Fig. 4b corresponds to this global (averaged) value. The local value of α seems to decrease at first, is stable between 1985 and 1991 and again between 1991 and 1995, but presents sharp variations at precise times. For example, around spring-summer 1990, the local value of α decreases abruptly and again in mid-1994. These events are probably to be associated with real political or economic events having an impact on international monetary policy.

Following the reported events in the booklets of OCDE [25], we have been able to interpret the last part of Fig. 4b. Between 1990 and 1995, the monetary strategy of the Bundesbank in Germany seems to have controlled the evolution of the exponent α. Indeed, we have noted the increases and decreases of the discount rate imposed by the german central bank during this period. These increases (decreases) are denoted in Fig. 4b by upward (downward) arrows. Surprisingly, the increases (decreases) of the discount rate imposed by the Bundesbank seem to imply increases (decreases) of the local value of α. These dated events corresponding to variations of both α and the discount rates are listed in Table 2. Such political data between February 1993 and February 1994 were not available to us. During that year, the trend in α is markedly negative (Fig. 4b). If the economic policy data becomes available, its relationship to the drop of the discount rate from 8% to 5.25% during this period will probably become clearer.

TABLE 2. Months and Years for both Bundesbank discount rate modifications, and the variations of the local value of α for the GBP/DEM ratio during the time span 1991-94. Data from OCDE, Brussels.

date	Bundesbank discount rate	α
Jan. 91	↑ 6.50%	0.43 ↑
Aug. 91	↑ 7.50%	0.50 ↑
Dec. 91	↑ 8.00%	0.54 ↑
Jul. 92	↑ 8.75%	0.55 ↑
Sep. 92	↓ 8.25%	0.58 ↓
Feb. 93	↓ 8.00%	0.57 ↓
Feb. 94	↓ 5.25%	0.55 ↓
Apr. 94	↓ 5.00%	0.50 ↓
May. 94	↓ 4.50%	0.49 ↓

A similar analysis has been performed on other time series in order to check the non-stationarity of α. The USD/DEM case was discussed in Ref.[10]. For all such analyzed ratios, the local α value decreases smoothly and seems to reach 1/2. This supports the idea that the foreign exchange market is actually governed by random conditions or - in the more usual terminology of economics - is *efficient* [6].

6. Robustness of DFA: Linear vs. Cubic Detrending

6.1. ARTIFICIAL SIGNALS

It could be rightfully asked whether the valleys and peaks observed in the time evolution of α are artefacts or not. In order to test the presence of artefacts, we have performed a Detrended Fluctuation Analysis with a non-linear detrending, i.e.

$$z(n) = cn^3 + dn^2 + en + f \tag{5}$$

on artificial self-affine signals in order to verify the robustness of the technique. Cubic detrending was used in order to avoid subtle effects which can occur e.g. in the presence of asymmetrically large fluctuations [13, 14].

The artifical time series used for the following demonstration within the successive random addition method originates in $d = 1$ landscape profile construction. This method is also called "midpoint displacement" in the literature [26]. With this iteration-based algorithm one generates a sequence of length $N = 2^i + 1$ where i is an iteration number. At each iteration, one find the intermediate positions (midpoints) of couples of neighboring points and calculates the values of the signal through some interpolation with respect to neighboring couples. The values of y on the midpoints are then displaced by random numbers chosen from a normal distribution with zero mean and variance $\sigma^2/2^{2iH}$. The parameter H is the Hurst exponent of the resulting self-affine signal or fractional Brownian motion.

Using the successive random addition technique described above, we have built time series of length $N = 4096$ ($i = 12$), i.e. the approximate size of our time series investigated in the preceding sections. For various signals $v(t)$ with different H values, we have measured the global exponent α.

Fig. 5 presents the relative error $(H - \alpha)/\alpha$. Both linear and cubic detrending are illustrated. As expected, we have found a global α value close to H. For H close to zero, the relative error of the DFA technique is larger than 30%. However, for $H > 0.3$, the relative error becomes quite small. A systematic difference between α values determined with linear and cubic detrendings seems to be present.

Local α values were estimated for these artifical signals using some observation window as described in Sec. 5. As expected, α is roughly constant and near H for these artificial time series.

6.2. REAL SIGNALS

We have also performed cubic detrending on the economic series and we have observed the same features (peaks and valleys) at the same dates as

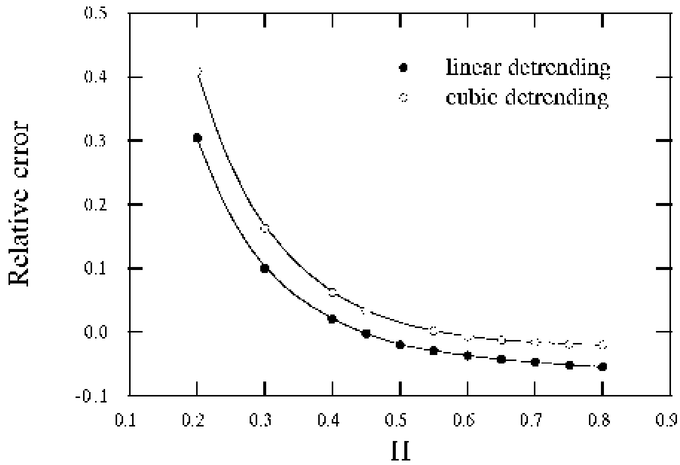

Figure 5. Relative error for the estimation of the measured α exponent as a function of the *input* Hurst exponent H of the artificial time series. Both linear and cubic detrending results are illustrated.

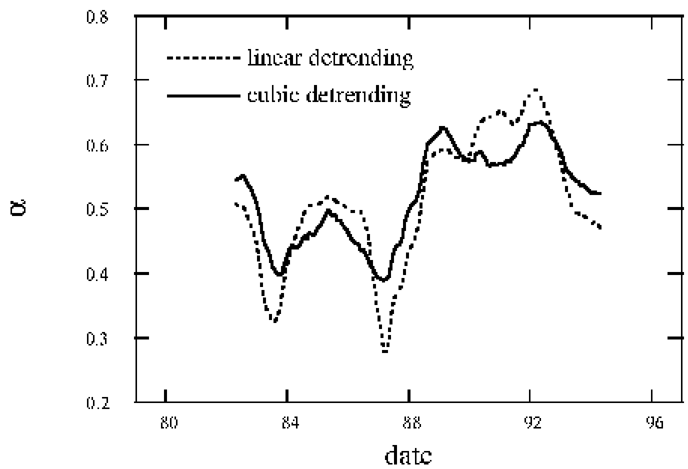

Figure 6. The local exponent α for the evolution of the USD/DEM exchange rate. Both linear and cubic detrending results are illustrated.

for the linear detrending as an example. Fig. 6 presents both linear and cubic detrendings for the USD/DEM ratio. No significant difference of the α behavior is observed for either detrending. Thus, peaks and valleys of the α exponent seem to be relevant and robust and can be trusted in economic signals.

6.3. ADDITIONAL REMARKS

This section is devoted to providing additional information to help beginners in using the DFA method.

First, the DFA algorithm is "easy" to compute. However, the probing of local correlations as well as non-linear detrending requires long computation times. Non-linear detrendings are, however, needed in some cases. Indeed, some events like a crash or an anomalous upsurge in stock markets can lead to undesired jumps of α, thereby screening some information. This is illustrated in Fig. 7 which presents the evolution of the Dow Jones Industrial Average [24]. The linear DFA result of the latter signal is presented in Fig. 8. On October 19th, 1987, the Dow Jones index dropped by more than 20% leading to a *singularity* in the data. We analyzed two time series: (i) the original data and (ii) the data transformed after deleting the singularity and rescaling (see the dashed line in Fig. 7). As observed in Fig. 8, the linear DFA performed on the original time series gives a huge jump of α around October 1987. This result suggests that the New York stock market became persistent in 1987. However, the results for the rescaled time series (the dashed line in Fig. 7) exhibit a small jump of α suggesting that the New York stock market was antipersistent in 1987. This remaining jump could be associated with a precursory or aftershock crash pattern [27, 28]. This observation is consistent with the antipersistent behavior taking place in 1986 and 1988. As a consequence of the large difference obtained in the results, we argue that an improved DFA method is necessary in order to judiciously quantify financial data.

A second remark is that the local exponent α as measured here corresponds to the center of the moving window since the different boxes are averaged *in* this window. One may ask where/when the value of α is really measured. We prefer to consider that α is not measured at the origin of the window as done in some other works [21]. This *a posteriori* remark seems very crucial for associating dated *political or economic policy* events with variations of the local exponent α. It can be shown that only when taking into account the above remark, can the α jump in Fig. 8 be directly associated with the date of the Dow Jones index crash.

7. Conclusion

The Detrended Fluctuation Analysis method has been hereby shown to be useful for analyzing the nature (persistency or anti-persistency) of economic fluctuations like those observed on daily currency exchange rates. Several cases were used and currencies could indeed be categorized into three different sets. These are associated with different economic and political conditions. It has been noted that known economic policy events, in

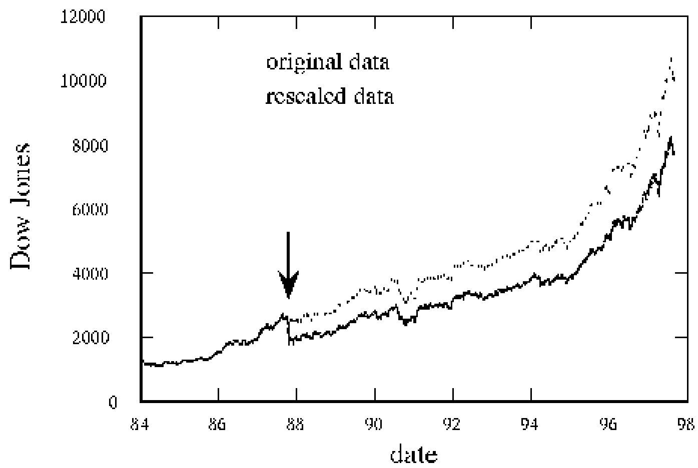

Figure 7. The daily evolution of the Dow Jones Industrial Average from January 1984 till August 1997. *Black Monday*, i.e. the October 1987 crash is denoted by an arrow. The continuous curve is for the original data, and the dashed line corresponds to the data for which the *Black Monday* singularity is deleted.

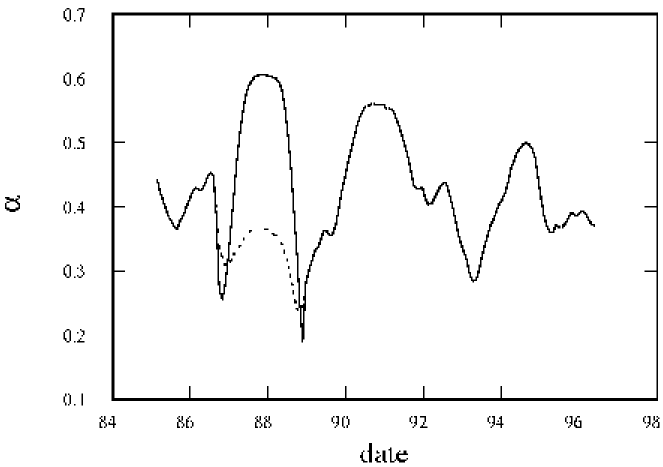

Figure 8. The local exponent α for the daily evolution of the Dow Jones Industrial Average from January 1984 till August 1997. Two results of the linear DFA are given: (i) the continuous curve is for the original data, and (ii) the dashed line corresponds to the data for which the *Black Monday* singularity is deleted.

particular through the leading central bank policies, have a signature in the α exponent characterizing the DFA technique.

We have tested whether the α exponent behaves differently depending on the detrending function. For the linear and the cubic cases which were examined, no significant difference was found, but other detrending functions might be tested for better extraction of the α value if necessary and on other signals.

Finally, we have tested the robustness of DFA over artificial signals and given some warning on the output date at which the α exponent is supposed to be measured. This resulted from a test of the Dow Jones Industrial Index in the vicinity of the October 19, 1987 crash.

8. Acknowledgements

This work was supported through a FNRS fellowship to NV. A special grant from FNRS/LOTTO allowed us to perform certain numerical tasks. The "Générale de Banque" of Belgium [18] provided the daily data. We thank M. Antoni, J.-P. Bouchaud, L. Jaeger, J. Kertész, A. Minguet, F. Schmitt, H.E. Stanley, and D. Stauffer for stimulating comments.

References

1. B.B.Mandelbrot, *The Fractal Geometry of Nature*, (W.H.Freeman, New York, 1982)
2. J.-F.Gouyet, *Physique et Structures Fractales*, (Masson, Paris, 1992) p.118
3. P.Bak, *How Nature Works*, (Copernicus, New York, 1996)
4. E.F.Fama, *J. Finance* **45**, 1089 (1990)
5. B.B.Mandelbrot, *J. Business* **36**, 349 (1963)
6. E.E.Peters, *Fractal Market Analysis*, (Wiley, New York, 1994)
7. M.H.R.Stanley, L.A.N.Amaral, S.V.Buldyrev, S.Havlin, H.Leschorn, P.Maass, M.A.Salinger and H.E.Stanley, *Nature* **379**, 804 (1996)
8. R.N.Mantegna and H.E.Stanley, *Nature* **376**, 46 (1995)
9. J.B.Ramsey, *Fractals* ; J.B.Ramsey and Z.Zhang, *J. Empirical Finance*, *in press* (1997)
10. N.Vandewalle and M.Ausloos, *Physica A* **246**, 454 (1997)
11. C.-K.Peng, S.V.Buldyrev, S.Havlin, M.Simmons, H.E.Stanley and A.L.Goldberger, *Phys. Rev. E* **49**, 1685 (1994)
12. H.E.Stanley, S.V.Buldyrev, A.L.Goldberger, S.Havlin, C.-K.Peng and M.Simmons, *Physica A* **200**, 4 (1996)
13. N.Vandewalle and M.Ausloos, in *Int. J. Comput. Anticipat. Syst.* **1**, 342 (1998)
14. N.Vandewalle and M.Ausloos, *Phys. Rev. E*, in press (November 1998)
15. J.Feder, *Fractals*, (Plenum, New-York, 1988) p.170
16. B.J.West and W.Deering, *Phys. Rep.* **246**, 1 (1994)
17. J.Beran, *Statistics for Long-Memory Processes*, (Chapman & Hall, New York, 1994)
18. J.Pirard and P.Praet, *private communication*
19. P.G.De Gennes, *Scaling Concepts in Polymer Physics*, (Cornell Univ. Press, New York, 1985)
20. J.-P.Bouchaud and A.Georges, *Phys. Rep.* **195**, 127 (1990)
21. Y.Liu, P.Cizeau, M.Meyer, C.-K.Peng and H.E.Stanley, *Physica A* **245**, 437 (1997)
22. N.Vandewalle and M.Ausloos, *Eur. Phys. J. B* **4**, 257 (1998)
23. S.Ghashghaie, W.Breymann, J.Peinke, P.Talkner and Y.Dodge, *Nature* **381**, 767 (1996)
24. Source: DATASTREAM
25. Office de coopération et de développement Economique (OCDE), Ministery of Economic Affairs, Brussels.
26. R.F.Voss, in *Fundamental Algorithms in Computer Graphics*, R.A.Earnshaw ed.

(Springer, Berlin, 1985) pp. 805-835
27. D.Sornette, A.Johansen and J.-P.Bouchaud, *J. Phys. I (France)* **6**, 167 (1996)
28. N.Vandewalle, Ph.Boveroux, A.Minguet and M.Ausloos, *Physica A*, **255**, 201 (1998)