ECONOMIC AND TECHNOLOGICAL SEARCH PROCESSES IN A COMPLEX ADAPTIVE LANDSCAPE

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1. Introduction

The analogies between economic and biological evolution are extensively discussed in the literature. Approaches that describe technological or economic change from a biophysical perspective include concepts such as competition and selection and corresponding mathematical models (Nelson *et al.*, 1976; Montroll, Shuler, 1979; Jiménez–Montaño, Ebeling, 1980; Nelson, Winter, 1982; Dosi *et al.*, 1988; Hanusch, 1988; Arthur, 1989; Witt, 1990; Saviotti, Metcalfe, 1991; Weidlich, 1991; Allen *et al.*, 1992; Karmeshu, 1992; Weidlich, Braun, 1992; Day, Ping Chen, 1993; Witt, 1993; Leydesdorff, van den Besselaar, 1994; Silverberg, Verspagen, 1994; Bruckner *et al.*, 1996; Kwasnicki, 1996; Saviotti, 1996; Karmeshu, Jain, 1997; Schweitzer, 1997). Self-organization theories from the physical research tradition including irreversibility, non-linearity and fluctuations have mainly influenced this direction. Recently, concepts like fractals, self-organized criticality and scaling originating in statistical physics have been successfully applied to economic problems, e.g., firm growth and financial market dynamics (Takayasu *et al.*, 1992; Sato, Takayasu, 1997; Mantegna *et al.*, 1995; Stanley *et al.*, 1996; Lubashevsky *et al.*, 1997). In this paper we describe the evolution of technological or firm populations as hill-climbing process in an adaptive fitness landscape over a continuous characteristics space. The relevance of the concept of an adaptive landscape for economic and technological evolution has been discussed, notably by Allen (1994; 1995). Following this line we develop in this paper a framework for continuous models of technological and economic evolution. We demonstrate the potential of such models, which in certain respects, goes beyond the widespread application of discrete replicator dynamics.

1.1. CONCEPTUAL BACKGROUND

A technology can be described by a large number of attributes, features or characteristics representing inherent technological aspects (performance, size, chemical composition) and economic parameters (input coefficients or certain product attributes) (Nelson, Winter, 1982). Often technological change is visualized by means of the temporal evolution of a single characteristic or some corresponding indicator. Some approaches extend such a one-dimensional description to a multidimensional mapping of technological evolution. Sahal (1981; 1985a) used a classification of design variables (e.g., the stroke length of an engine) and performance variables (e.g., the fuel consumption) to build a "topography of technological evolution". Based on this concept technological models occupy different loci in parameter space. The probability distribution of certain parameters forms a surface over the space, and the movement of the occupied regions indicates technological change. Saviotti and Metcalfe (1984) elaborated a methodological framework for a characteristics space of product technologies. They use two sets of output indicators, viz. technical characteristics and service characteristics, and describe processes like substitution, specialization and innovation in terms of changes in these parameter sets (see also (Saviotti, Bowman, 1984; Saviotti, 1985; Saviotti, Mani, 1995)).

A similar description may be developed for the evolution of firms and, in particular, their evaluation on the financial market. Indeed, the price of a firm's shares on the stock market depends on a multitude of characteristic features such as the capital stock, the investment rate, the management, the working skills, the technological profile and R&D strategies. The assessment of a firm through the market environment depends not only on its inherent properties, but also on its interactions with other firms (e.g., competitors, suppliers) and on the general properties of economic and political networks like information flows.

In this paper we develop a framework of continuous evolutionary models combining the characteristics representation of a firm or a technology with an additional dimension corresponding to a valuation or fitness function in evolutionary dynamics. This dynamics incorporates various interactions between the participating competitors and feedback into the valuation function. In the following we propose the model framework mainly in the context of technological evolution. We want to emphasize, however, that the model framework itself is more universal and has a broad spectrum of applications. Therefore, an application of the modeling framework to market dynamics is also proposed. Some possible implications of such type of models for the relation of firm behavior and financial markets are brought out in the conclusions.

Following the above mentioned concepts for mapping technological evolution we assume that the characteristics space is formed by three sets of indicators: technical characteristics X_1, X_2, $X_3, \ldots, X_l$, service characteristics Y_1, Y_2, $Y_3, \ldots, Y_m$ and financial characteristics Z_1, Z_2, $Z_3, \ldots, Z_k$. Then, each product model is represented by a point in this space described by a certain vector $\vec{q} = (q_1, q_2, \ldots, q_d) = (X_1, X_2, \ldots, X_l, Y_1, Y_2, \ldots, Y_m, Z_1, Z_2, \ldots, Z_k)$ with $d = l + m + k$. All real vectors span the characteristics space Q which is a real Euclidean vector space. This space can be regarded as the analog of phenotype space in biology. Obviously, only a subset of all possible combinations of real numbers q_i is realized in the real evolutionary process. Most of the regions of Q are empty. Some combinations of parameter values are self- contradictory (see e.g., (Foray, Grübler, 1990)). Therefore, the set of possible technological populations Q is restricted to certain compact areas of Q.

Next we define a population density $x(\vec{q}, t) = x(q_1, q_2, \ldots, q_d, t)$ as a function over the characteristics space. Morphologically oriented investigations of technologies show that the dominant or the finally established technology is typically selected from a variety of possibilities (Foray, Grübler, 1990; Durand, 1992). In general, the value of the function stands for the number or frequency that a certain product model is realized (more precisely, $x(q)dq$ stands for the number of product models in the interval dq while $x(q, t)$ is the corresponding population density function). A more precise definition will affect the focus of the competition and selection considered. Here, we refer to the number of commodities produced and sold as a quantitative expression of population density. In this case, following Dosi (1982), competition is clearly realized through the market. Further, in order to understand the movement of technological populations in the characteristics space and to follow their trajectory, especially between locations which are able to compete, it seems to be useful to also include models that have been proposed but not yet market-proven. In light of this phenotypic approach the realized technological populations result from a competition and selection process through which the shape of the population density in terms of its modality changes. The occurrence and movement of such relatively stable populations corresponds clearly to concepts of *technological regimes* and *natural trajectories* (Nelson, Winter, 1977), *dominant design* (Utterback, Abernathy, 1975; Abernathy, Utterback, 1978), *technological guideposts* (Sahal 1981, 1985b) and *technological paradigms/technological trajectories* (Dosi, 1982). These approaches are relevant in more than one respect for the modeling framework developed in this paper.

Further utilizing the analogy with biophysical evolution, we assume that the dynamics of competing economic or technological populations corresponds to a fitness function that spans a landscape over a multidimensional characteristics space. The success of a firm or technology is defined in economic terms through market environments (Nelson, Winter, 1982). Defining the characteristics of the external environment in a quantitative way seems to be extremely complex. Moreover, theory alone cannot tell us the exact structure of this fitness landscape; however, from existing economic/technological populations we can infer some of its characteristics. Looking at the multiplicity of technologies, the landscape, globally considered, will clearly be multimodal. Even in a certain restricted region of similar characteristics, specialization or the existence of niches can also be understood as a sign of multimodality. Further, we can assume that changes in the landscape will be smooth. Besides multimodality there are reasons to believe that the landscape has a very complicated structure. One reason is that economic structures and technologies normally represent compromises between several needs. Finding the optimal solution under conflicting conditions is a so-called “frustrated problem” with a chaotically shaped landscape (Anderson, 1983). However, using recently developed mathematical techniques, it is possible to draw qualitative conclusions about the characteristics of evolutionary processes under such conditions.

In this paper we use the conceptual setting developed to implement a continuous evolutionary model framework as a new approach to economic/technological evolution. If we see fitness as an unknown landscape over characteristics space, economic and technological change is mainly a search and learning-by-doing process. This is quite consistent with most of the evolutionary theories of technological change and economic development. By analogy with phenotypical evolution in biology, we describe the search process for economic and technological improvements as hill-climbing process in an adaptive evolutionary fitness landscape. In biophysics this type of model, originally proposed by Wright (1932) and Conrad (1983), was mathematically developed by Feistel and Ebeling (1989) (see also (Ebeling *et al.*, 1990)).

The central point of the theory of the adaptive evolutionary landscape is to link the growth and movement of populations to changes in the fitness. This feedback can lead to an increase in the survival possibilities for some parts of the population but to a decline for others. In technological evolution the exclusive character of technological paradigms (or lock-in) (Arthur, 1988) seems to indicate such behavior.

2. Models of a Search Process in Complex Adaptive Landscapes

A formal model of an evolutionary search in a phenotype space reads in a continuous formulation (Ebeling *et al.*, 1984; Feistel, Ebeling, 1989; Ebeling *et al.*, 1990):

$$\partial_t x(q,t) \;=\; x(q,t)\, w\,(q;\{x\}) \;+\; Mx(q,t) \tag{1}$$

where w is a function of q, and possibly also a functional of $x(q)$. M is the mutation operator. The function w describes the rate of growth or decline of the population density $x(q,t)$ (e.g., output of product models). In biophysical models w stands for the process of self-reproduction (replication). The "reproduction rate" $w(q)$ defines an evolutionary landscape superimposed on a high-dimensional space Q.

2.1. THE FISHER-EIGEN MODEL OF TECHNOLOGICAL EVOLUTION

In the rather simple Fisher-Eigen model $w(q)$ is given as the difference between a self-reproduction function $E(q)$ and its (only time-dependent) ensemble average (Feistel, Ebeling, 1982):

$$w(q,t) \;=\; E(q) \;-\; \langle E\rangle. \tag{2}$$

Then, the linear functional $w(q;x)$ is given as:

$$w \;=\; E(q) \;-\; N^{-1}\int dq'\; E(q')\; x(q') \tag{3}$$

where

$$N \;=\; \int x(q',t)\; dq'$$

is the overall population size. The population average $\langle E\rangle$ is defined by:

$$\langle E\rangle \;=\; \frac{\int dq'\; E(q')\; x(q',t)}{\int dq'\; x(q',t)}. \tag{4}$$

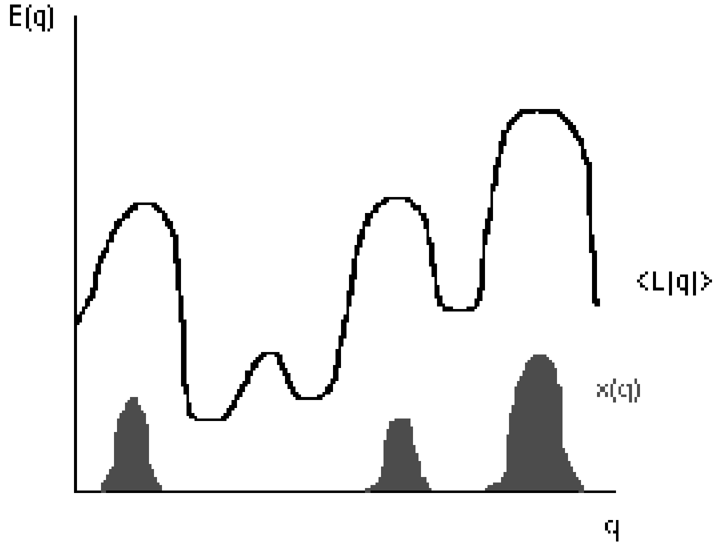

Figure 1: One-dimensional representation of a fitness landscape and locations of technological populations

The integral is extended over the whole of phenotype space. Since the shape of the fitness functional will be unknown in general, we assume that $E(q)$ (and consequently $w(q)$) has the form of a correlated random landscape. Then, in analogy with solid state physics we may consider $E(q)$ as a "correlated random potential". Several specific features of the search in high-dimensional landscapes ($d \gg 1$) have been analyzed by Conrad and Ebeling (1992). In particular, it has been shown that in high-dimensional correlated landscapes the probability that a given stationary point is a saddle point increases with the dimension. In other words, the searcher (or searching population) is more often confronted with the problem of leaving a saddle point than of escaping from a proper relative maximum.

The second term in (1) denotes a mutation operator. In the simplest case it may be modeled as a diffusion term:

$$Mx(q,t) \;=\; D\Delta x(q,t) \tag{5}$$

where Δ is the Laplace operator and D is the diffusion coefficient. In a more general form we introduce the diffusion matrix D_{ij}

$$Mx(q,t) \;=\; \sum_i \partial_i \sum_{j=1}^{d} D_{ij}\partial_j x(q,t). \tag{6}$$

In a still more general setting M denotes a linear operator

$$Mx \;=\; \int \left[A\left(q,q'\right) x\left(q',t\right) \;-\; A\left(q',q\right) x\left(q,t\right) \right] .dq' \qquad (7)$$

Here the transition matrix may also take into account long range transitions (as e.g. Cauchy or Levy-type transitions).

So far, a full analysis is available only for the simplest case given by Eqs. (1), (3) and (5) (Feistel, Ebeling, 1989; Ebeling *et al.*, 1990). The dynamic properties of equations of this type were investigated by Zeldovich *et al.* (Zeldovich *et al.*, 1985) and others (Engel, Ebeling, 1987; Asselmeyer *et al.*, 1996). Mathematically speaking, they are closely related to the Bloch equations of statistical physics (Schrödinger equations with imaginary time). The problem reduces more or less to the solution of a Schrödinger eigenvalue problem for the potential $U(q) = -E(q)$. It may be considered as a great advantage of this kind of model that many results from quantum mechanics for more or less complicated potentials are available.

As shown in Figure 1, in the Fisher-Eigen approach the self-reproduction function $E(q)$ can be considered as a landscape over the characteristics space. The region of technological populations follows the maxima of this function which lie above the ensemble average. The fitness landscape $E(q)$ is static, i.e. time-independent. Moreover, it does not depend on the interaction between the members of the populations. The more general function $w(q)$ undergoes temporal changes only through the ensemble average $\langle E(q) \rangle$. Models of this kind can be used to understand, e.g., concentration processes and the relation between continuity and discontinuity in technological evolution.

For the problem of economic and technological evolution it seems to be very interesting to consider the case when changes of the fitness landscape are endogenously determined and are related to the movements of the populations themselves. "In changing the relative significance of competing technologies, selection also results in changes in the price structures that evaluate performance characteristics, so reshaping the selection environment. Indeed, one of the central themes of the evolutionary approach to competition is that technologies and their selection environments coevolve." (Metcalfe, Gibbons, 1989, p. 157). The dependence of the fitness landscape on growth and movement of populations is central to the theory of the adaptive evolutionary landscape. We will overcome the disadvantage of the models considered so far for modeling this kind of feedback in more advanced models exhibiting co-evolution of the competitors.

2.2. LOTKA-VOLTERRA DYNAMICS OF TECHNOLOGICAL EVOLUTION

Such a more flexible dynamics results if $w(q)$ is considered as a linear functional in the following way

$$w\left(q; x(q,t)\right) \;=\; a(q) \;+\; \int b\left(q, q'\right) x\left(q', t\right)\; dq'. \tag{8}$$

The special case

$$a(q) \;=\; E(q), \quad b\left(q, q'\right) \;=\; -E\left(q'\right) \tag{9}$$

brings us back to the Fisher-Eigen case (3). In the general setting we find non-linear equations of the type

$$\partial_t x(q,t) \;=\; \left[a(q) \;+\; \int b\left(q, q'\right) x\left(q', t\right)\right] x(q,t) \;+\; Mx(q,t). \tag{10}$$

The dynamics of this type of equations, which resemble the Hartree equations with imaginary time, has not been thoroughly investigated; only the initial steps were carried out in (Feistel, Ebeling, 1989; Ebeling *et al.*, 1990). The first important result for the case $D = 0$ is due to Musher *et al.* (1995) in connection with the theory of weak Langmuir turbulence. Zakharov proposed recently modelling complicated nonlinear evolution processes including economic and social processes using equations of the type of Eq. (10) (Zakharov, 1995). In particular, it can be shown that nonlinear growth rates as modeled by Eq. (10) lead to solutions with a self-accelerating character (Ebeling *et al.*, 1990; Bruckner *et al.*, 1989). This entails an effect which is sometimes called the Matthew effect: *The rich become richer and the poor become poorer* or the principle of cumulative advantage (Merton, 1968; Bonitz, 1997; Bonitz *et al.*, 1997).

3. Technological Trajectories and Continuity versus Discontinuity in the Process of Technological Change

To demonstrate the capabilities of such models we consider in more detail the case of Fisher-Eigen dynamics. As a starting point we use the following equation:

$$\partial_t x(q,t) \;=\; x(q,t)\left[E(q) - F(t)\right] \;+\; D\Delta x(q,t). \tag{11}$$

This is a slightly generalized form of Fisher-Eigen dynamics (see Eq. (1) and Eq. (2)) because $F(t)$ is not identical to the population average of $E(q)$. Further, the mutation operator is modeled with a diffusion term (cf. Eq.

(5)). We now compare Eq. (11) with the famous replicator model due to Fisher-Eigen:

$$\frac{d}{dt} x_i(t) \;=\; (A_i - D_i)\, x_i \;+\; \sum_j (A_{ij}\, x_j - A_{ji}\, x_i) \;-\; F(t)\, x_i \qquad (12)$$

$$i \;=\; 1, \ldots, n.$$

Here, A_i is the self-reproduction rate, D_i is that for decline processes and $F(t)$ includes boundary conditions like a constant overall population. The summation term includes error reproduction or mutations which in the case of social systems can be understood as transition or exchange processes (Bruckner *et al.*, 1989; Bruckner *et al.*, 1996).

In the continuous model framework the typological classification of populations used in discrete replicator models is replaced by a characteristics representation. Subsequently, the population density x_i is replaced by the function $x(q,t)$ in a continuous phenotype-like space. If the population density is concentrated in certain regions of Q ("islands") then these "islands" can be related to the original classified populations. The "selective value" $E(q)$ is linked to the net reproduction rate $(A_i - D_i)$. The choice of a diffusion-like mutation operator (cf. Eq. (5) and Eq. (11)) corresponds to the assumption that the mutation rates A_{ij} are symmetric, homogeneous and of short range. For technological evolution - and for social processes in general - it seems to be of particular interest to consider inhomogeneous and "directed" mutations. The emergence of a new technology in the system is related to a stepwise process ranging from research at the "pure science"-level, to applied R&D level to production relevance. The final introduction of a new technology, understood as a mutation in the system of established technologies, is the result of a multi-level process. Different institutions are the carriers of these processes (Freeman, 1974). At each level decisions about selection between variants are taking place (Dosi, 1982). These are influenced by feedback mechanisms. This kind of "contextual pre-selection" can be understood as "selection of the 'mutation generating' mechanisms" (Dosi, 1982) and can be modeled by means of more elaborate mutation operators. Here, for the sake of mathematical simplicity we restrict ourselves to the diffusion approach.

In contrast to discrete models, the vanishing, merging, division and emergence of technologies are expressed by changes in the shape of the function $x(q)$, without having to consider changes in the taxonomy of the model. This results in a greater mathematical complexity of the model.

As mentioned above, the population density follows the shape of the fitness landscape. If we assume $E(q)$ to be a random function, then the shape of $x(q,t)$ is sensitive to statistical properties of this function given by the probability density functional $P[E(q)]$.

In Eq. (11) the term $x(q,t)[E(q)-F(t)]$ describes the selection process. This becomes evident if we consider the temporal evolution of populations without mutations:

$$\partial_t x(q,t) \quad = \quad x(q,t)\,[E(q)-F(t)] \tag{13}$$

For the Gaussian distribution $P[E(q)]$, this was investigated in (Zeldovich *et al.*, 1983) using percolation theory. The result shows that with increasing time the population is concentrated in islands which correspond to particularly high values of the random function $E(q)$. These islands of high density are surrounded by regions of low density. This means that the selection process leads to a concentration of the distribution around the maxima. The diffusion process, on the other hand, leads to a widening of the distribution and extending of its tails.

The mathematical solution of Eq. (11) in the presence of mutations is more complicated. Using the analogy with the Schrödinger equation for an electron in a random field some approximate expression for the time dependent solution $x(q,t)$ was given by Ebeling *et al.* (1984) (see also (Feistel, Ebeling, 1989; Ebeling *et al.*, 1990)). It can be shown that the existence of technological trajectories and technological populations correspond to the problem of the existence of localized states (according to the localization problem in random potentials). A localized state can be understood as a distribution of product models around a dominant design belonging to a single technological population. Technological trajectories can be understood as being formed by the movement of the localization centers. It is well known that for high-dimensional spaces ($d \geq 4$) and δ-correlated potentials $E(q)$, there are no localized states at all. Therefore, the existence of a correlation length greater than zero seems to be a necessary condition for the emergence of distinguishable parts (or populations) in the population density function. This is in accordance with the smoothness postulate of an adaptive landscape formulated by Conrad (1978; 1983). An evolutionary system can only develop a strategy for search processes in a landscape with correlations. Furthermore, this entails that we have to distinguish between lack of information about the environment (which can be modeled by means of a random fitness function with certain statistical properties) and complete irregularity (stochasticity) where in principle no extrapolation from the local knowledge is possible. From the existence of localized states in a random unrestricted extension of the function $E(q)$ (resp. $w(q)$) the following statement for the time evolution of $x(q,t)$ can be made: with increasing time the density becomes concentrated in localized states with decreasing localization radi (for mathematical details see (Feistel, Ebeling, 1989; Ebeling *et al.*, 1990)). This process can continue endlessly.

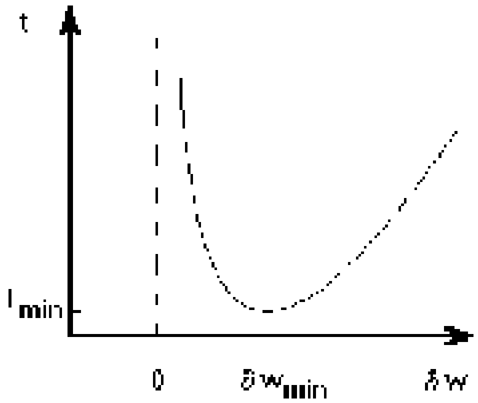

Figure 2: Schematic representation of the transition time as a function of the target fitness value

The concentration of populations in regions of the fitness landscape with high valuation is one effect. For the continuation of evolution it seems to be very interesting to ask about possibilities to abandon such regions for new ones. Interesting characteristics of this transition process are the mean distance between the starting point q' and the nearest localization center, and also the mean transition time between successive steps. The most important result of the mathematical analysis (Ebeling *et al.*, 1990) is, that there exists a characteristic finite jump – the so-called "evolutionary quantum".

If we consider the transition from one localized state around a fitness maximum w_0 to the next nearest state with another maximum w_n, the transition time t as a function of the value of δw, has a specific form (see Figure 2). The existence of a minimum t_{min} can be understood from the dynamics of the system. Transitions to much higher maxima of w require several successful mutations. Such big jumps are relatively rare. This is expressed by high values of the transition time for $\delta w \gg \delta w_{min}$. On the other hand, shifts to maxima with similar fitnesses can be achieved much faster by mutations, but the following selection process needs more time because of the small improvement of the new area (population). This means that the transition time also increases for $\delta w \ll \delta w_{min}$. Therefore, δw_{min} can be understood as an optimal step of improvement - also called the "quantum of evolution". In characteristics space this discontinuity corresponds to a step-like behavior.

Evolution in correlated valuation landscapes proceeds in a jump-like fashion. This means that longer periods of smooth evolution of technologies are sometimes interrupted by jumps. Therefore, "incremental" and "radical" innovations are both part of the system dynamics driven by mutations. From the perspective of the system "big jumps" occur as the radical innovations changing the composition of the system. The drift stands for continuous change (incremental innovations). It was shown by Zhang, Engel and one of the present authors (Zhang, 1986; Engel, Ebeling, 1987) that there exists a definite scaling between the distance $|\delta q|$ of the jump in the characteristics space and the characteristic time τ for the jump

$$|\delta q| = \frac{\tau}{[\ln \tau]^{1/2}}. \tag{14}$$

4. Applications to Market Dynamics

The dynamics of the market is central to the competition and selection processes in the real economy. In spite of the fact that the real dynamics is extraordinarily complicated let us try to model typical features with the tools developed above. So far, in the context of technological evolution we have considered the market as a selective environment. Now we consider the market itself in more detail. First we assume the market to be an environment of a technological or economic system and assign the fluctuating properties to the market itself. We start with the Fisher-Eigen equation. As mentioned above, the determination of the valuation function is a difficult question. So far, we have modeled principal uncertainty in terms of spatial fluctuations of the valuation function over characteristics space. Now we introduce time-related fluctuations. In some earlier work (Feistel, Ebeling, 1989; Ebeling *et al.*, 1990) the hypothesis was developed that the replication rate of technological populations is proportional to the profit which a particular technology can generate in the market

$$E(q,t) = \text{const. } Pf(q,t). \tag{15}$$

Here, the profit $Pf(q)$ is determined as the difference of the price $P_c(q)$ prevailing in the market and the production cost $C(q)$. This leads to:

$$E(q,t) = \text{const. } \left(P_c(q,t) - C(q,t)\right), \tag{16}$$

and in the simplest case $F(t)$ is determined by the mean

$$F(t) = \text{const. } \frac{\int dq\; x(q,t)\,[P_c(q,t) - C(q,t)]}{\int dq\; x(q,t)}. \tag{17}$$

If we consider firms as carriers of the evolutionary process (cf. (Bruckner *et al.*, 1996)) then $E(q,t)$ can be understood as the net growth rate of firms with certain characteristics q (including the technology in use). In this case the characteristic space has to be re-constructed conceptually in terms of the characteristics of firms. According to recent findings (Amaral *et al.*, 1997) we may assume that the company growth rate consists of a systematic part and a fluctuating part

$$E(q,t) \;=\; E(q) \;+\; \delta E(q,t) \tag{18}$$

where the fluctuations obey certain scaling rules (Amaral *et al.*, 1997).

In another extension of the model framework being discussed we will consider the market itself as an evolving system. The interplay of sellers and buyers in the stock market serves as an example. So far, our analysis was entirely concentrated on Fisher-Eigen dynamics. Much less work has been devoted to economic applications of the continuous Lotka-Volterra equation, in spite of numerous applications of its discrete counterpart (cf. (Levy, Solomon, 1996)). As far as we see, continuous Lotka-Volterra equations are quite appropriate to model the market dynamics expressed by the interplay between buyers and sellers. Recently, Takayasu *et al.* (Takayasu *et al.*, 1992; Sato, Takayasu, 1997) have succeeded in describing stock market dynamics by discrete stochastic models. In the Sato-Takayasu model dealer i on a stock market is characterized by a minimum price B_i and by an interval $L_i = \delta B_i$, giving the selling price $S_i = B_i + L_i$. If he succeeds to find another dealer in the market with the buying price B_j such that

$$B_i \;<\; B_j \;\leq\; B_i + \delta B_i \tag{19}$$

he can sell his product and make a profit. In case

$$B_j \;<\; B_i \;\leq\; B_j + \delta B_j \tag{20}$$

dealer i may buy a product from dealer j. We are not going to describe the details of the Sato-Takayasu model, which includes stochastic elements and describes correctly the fluctuations observed in the stock markets.

A plausible continuous variant of the Sato- Takayasu model may be formulated as follows. The interplay of buyers and sellers changes the amount and the distribution of money in the system. The amount of money forms a population over a space of characteristics of financial products. For each point of this space we define buying prices $B(q)$ and price intervals $L(q)$. Then the market dynamics is described by the Lotka-Volterra equations (Eqs.(5) and Eq.(10)) with a correlated random growth function $a(q,t) = E(q,t)$. The act of selling and buying is consequently expressed in the exchange term $b(q,q')$ which has to be antisymmetric. This kernel should be a functional of $B(q)$ and $L(q)$. A possible choice for the kernel is the derivative of a Gaussian

$$b\,(q, q') \;=\; \frac{1}{\tau}\;\frac{B(q) - B(q')}{\sqrt{L(q)}}\;\exp\left[-\frac{(B(q) - B(q'))^2}{2L(q)^2}\right] \qquad (21)$$

Then the dynamics works out as follows. If a dealer $B(q)$ succeeds in finding a partner with a higher buying price $B(q')$ such that $B(q) \leq B(q') \leq B(q) + L(q)$ he may sell his product and has a chance to grow. Otherwise, he still has a chance to buy from a dealer in the corridor below his own buying price and the possibility to sell the asset in the next turn.

5. Conclusions

In this paper a modeling framework for the co-evolution of economic or technological competitors in a continuous phenotype-like space is developed. The introduction of a characteristics space allows us, in principle, to distinguish between drifts and jumps in technological progress. Furthermore, the emergence, vanishing, differentiation or merging of different economic structures or technologies can be described. One purpose of the present paper is to demonstrate the versatility of continuous models for such processes. In this paper, the Fisher-Eigen dynamics and the Lotka-Volterra dynamics are introduced. In both cases there are different kinds of feedback mechanisms between the population density and the shape of the fitness landscape.

Concerning technological evolution, the adaptive landscape concept seems to be important for the understanding of an "innovative environment" (Dosi, Metcalfe, 1991) and the relation between "blind" and "directed" search strategies or the nature of change (Allen, Lesser, 1991). "Irreversibility is not only the result of imperfect information and sequentiality of decisions but is due to the fact that the world genuinely changes and it changes as a consequence of the very actions of the agents." (Dosi, Metcalfe, 1991, p. 146). The adaptive landscape concept is related to a Lotka-Volterra dynamics which expresses the mutual dependence of changes in the population density and changes in the fitness landscape. In as much as the fitness functional w depends both on the characteristics values q and the population density $x(q, t)$, the following circle can be observed: self-reproduction according to the fitness value leads to a change in the population density, this influences the shape of the fitness and, therefore, the self-reproduction. In this way, self-referentiality is introduced in the model.

A second objective of the present paper is to link the concept of an adaptive landscape to complex search processes. Therefore, the structure of the fitness landscape and, in particular, its different statistical properties must be considered in greater detail. The fitness function is assumed to be a correlated random potential. Thus links to mathematical techniques used in statistical physics can be established. In this paper some implications from recent research trends for technological evolution are discussed. In particular, for the Fisher-Eigen dynamics the occurrence of localized states and the jump-like character of evolution have been considered. On a short time scale the hill-climbing character of evolution due to the interaction of selection and diffusion processes leads to the development of an island structure in characteristics space. This corresponds to empirically observable technological populations. In the continuous approach the existence of such populations is not an initial assumption (like in discrete descriptions) but the result of search processes in a random landscape. On a long time scale hill-climbing proceeds by small but discrete steps. Transitions from an initial region with a certain given fitness w_0 to regions whose fitness is better by exactly one "quantum" are preferred. Surprisingly, this discontinuous character of evolution is the result of a continuous approach. This indicates that the stepwise character of technological evolution as observed is not a direct consequence of the discreteness of mutations but rather a general feature of the selection-mutation processes. Concerning the relation between incremental and radical innovations in the model framework developed here, shifts can be explained in terms of diffusion as well as jumps in terms of relocation of populations. The occurrence of an optimal step-width of improvement is probably related to the phenomenon of discrete steps of technological change described in the literature as the apparent paradox of a " 'discrete continuum' of technological change" (Durand, 1992). Furthermore, a scaling behavior between distance and the characteristic time of jumps can be observed.

Thus, the modeling framework developed here seems to be useful for the description of economic and technological systems which behave in a complicated unknown environment. The units of evolution can be described by a set of varying characteristics. In this paper we mainly consider an approach which describes technological evolution in terms of the movement of technological populations. In this case, product models or technological processes are taken as carriers of the evolution process. Another widespread approach to technological evolution considers the firms themselves as carriers of the search process for technological improvement and market success (Nelson, Winter, 1982; Allen, Lesser, 1991; Bruckner *et al.*, 1996). In such approaches and models, characteristics of firms such as size, age, belonging to a sector or technology, capital, etc. appear as variables, parameters and classification features. A possible extension of the model framework is the inclusion of temporal fluctuations in the valuation function (fitness landscape). Therefore, links to observed fluctuations in firms' growth rates can be established. According to the framework developed above, we can imagine that a particular choice of such economic, financial, organizational and technological characteristics can be used to construct a characteristics space in which firms occupy different regions, and populations are built by groups of firms with similar characteristics. Evaluations of these populations are performed through market environments. The stock market can be seen as a special case. The price of the shares of a firm on the stock market depends on a multitude of its characteristic features, but also on the interactions between the firms and on general properties of economic and political networks like information flows. The concept of evolution in an adaptive landscape can probably be used as an instrument to describe interactions between the firms' strategies and the reactions in the stock market. In this paper we have considered as an example the relevance of Lotka-Volterra models to stock market dynamics. In particular, a continuous formulation of the Sato-Takayasu model has been introduced.

In this paper we hope to have given some starting-points for further research on learning, problem solving and adaptation in economic and technological evolution, leading to a mathematical formulation of market dynamics.

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