A MICROSCOPIC MODEL OF THE FOREIGN EXCHANGE MARKET

R. CHATAGNY AND B. CHOPARD
Group for Parallel Computing, CUI
University of Geneva, Switzerland

1. Introduction

Economic processes are difficult to describe and understand because there is no theory explaining them in the same way as there is for phenomena appearing in the physical sciences. Yet economic processes play a crucial role in our everyday life and there is a great deal of effort devoted to predicting future trends and understanding how various parameters affect them.

The simulations that appeared in the course of the last couple of years [1, 2, 3, 4] made use of simplifying assumptions that prevented them from unravelling the full complexity of real financial markets. The aim of this paper is to present a new type of simulation closer to reality. The main idea is derived from the investigation of a wide range of complex physical systems, characterised by the fact that their macroscopic behaviour emerges from the collective action of many nonlinear agents in mutual interaction (lattice gases, etc.). We believe that a model reproducing the behaviour of the real actors on the market (traders, market makers, ...) can best simulate such a financial market. We hope that this simulation will help us improve our understanding of market mechanisms. Our simulation has been developed so that it may be used as an experimental laboratory mimicking the real market. In this sense, new trading strategies can be tested, the influence of the presence or the absence of the long-term traders can be measured and so on.

The signature of an economical process is often given by a time series (for instance the exchange rate between two currencies as a function of time, or the value of some financial index on the stock market). The observed behaviour looks rather chaotic and intricate, but a more careful analysis reveals some underlying statistical laws describing the data distribution as well as some scaling properties. To validate our simulation, we will compare these properties and distributions with those obtained in our artificial market. These scaling properties are also an indicator that some generic behaviour does emerge from the complicated interactions between the components of such a system. In particular, scale invariance shows up, indicating that the time resolution at which the system is observed is a crucial ingredient of the dynamics. This shows the soundness of our approach and justifies the idea that one has to introduce different time scales for the traders for example to be realistic in the simulation and to have the hope of seeing the emergence of the macro-structure of the market from its micro-structure (the actors of the market).

We shall focus our simulation on a specific financial system, the so-called Foreign Exchange (FX) Market. We first review its main rules and look at the way it differs from other financial markets. Then we explain the principles of our simulation and describe the behaviour of each actor in the market. The numerical simulations show that this model captures several of the qualitative features of the real market which arise from the interaction of many actors. Finally, we comment on how we could improve our results.

2. The Foreign Exchange Market

The FX market is slightly different from the other financial markets (the stock market, for instance) because there one does not exchange a valuable against a sum of money, but one currency for another currency. Every transaction price involves a pair of currencies, e.g. USD/DEM, USD/JPY or GBP/CHF[1]. The FX market is the most active of the financial markets with a daily turnover of more than 1000 billion USD, which is more than the total non-gold reserve of all industrial countries [5]. This is a consequence of the volume of money involved in each transaction and the large number of actors trading on this market.

In fact, these transactions only involve certain major currencies (USD, DEM, JPY, GBP and CHF). The USD is evidently the most important currency, being involved in 80% of the transactions. Only those rates where one of the currencies is the USD produce really active markets. This is the case for the USD/DEM rate which represents 20% of the activity of the FX market. For an indication of the level of activity, consider that the frequency of new rates on the USD/DEM market can reach one quotation every 3 or 4 seconds. Currencies other than the five major ones are quite negligible in comparison, and mainly concern local financial markets.

[1]Here we use the standard and obvious abbreviations of the International Organization for Standardization.

Contrary to other financial markets, the FX market is not attached to a particular physical location. For the stock market an asset is restricted to a financial location, for instance the New York Stock Exchange. On the FX, however, each pair of currencies can be theoretically traded in any financial location around the world. This implies that the FX market is a 24 hour global market, because a trader can always find an open financial location somewhere, so there are no opening and closing times.

Despite the fact that the FX market is continuous, its activity is not homogeneous in time and this causes difficulties when analyzing the data. The weekends are periods of very low activity. Daily variations can also be observed, due to the continental components corresponding to the three major financial regions: East Asia (Tokyo), Europe (London) and America (New York). Early in the night (GMT), the activity is low, because only the Asian financial markets are open. In the afternoon, the market is very active due to the transactions of the European and the American traders. The overlap of activities of these regions produces a regular pattern observed in the daily activity of the FX market. These fluctuations imply a non-stationary evolution of the exchange rates. To obtain a stationary signal and facilitate the study of the statistical properties of the FX market, it is customary to change the scale of time as a function of the activity of the markets [6].

Before we continue with explaining the microstructure and the workings of the FX market, let us define some basic macroscopic indicators of this market and some of their properties which we will subsequently use.

2.1. EXCHANGE RATE AND MARKET PRICE

Let us assume we have an amount s_A of currency A. This amount of money can be changed to another currency, say currency B. The exchange rate λ_{AB} from currency A to currency B is defined as $s_B = \lambda_{AB} s_A$, where s_B is the amount of currency B one receives from such a transaction.

λ_{BA} is the exchange rate for the inverse operation transforming the amount s_B into an amount s'_A of currency A, such that

$$s'_A = \lambda_{BA} s_B = \lambda_{BA} \lambda_{AB} s_A. \tag{1}$$

Clearly, one should have $s'_A < s_A$ if we use simultaneous exchange rates, otherwise one could produce money at no cost. Nevertheless it can sometimes happen that $s'_A > s_A$. This situation is called an arbitrage position and it has a very short lifetime. It disappears as soon as one discovers it. So in the normal case, we have

$$\lambda_{BA} \lambda_{AB} < 1. \tag{2}$$

This relation expresses the market "friction". We shall later show its importance for one type of agent in the market, the market makers.

Usually, the *bid* and *ask* prices (buying and selling prices) are used instead of λ_{AB} and λ_{BA}. They are defined as follows. First a reference currency is chosen, say currency A. The bid price at time t, $p_{bid}(t)$, is the amount of currency A that one would receive in exchange for one unit of currency B, and inversely $p_{ask}(t)$ is the amount of currency A that one has to give to obtain one unit of currency B.

From this definition we have that

$$p_{bid} = \lambda_{BA} \qquad \text{and} \qquad p_{ask} = \frac{1}{\lambda_{AB}}. \tag{3}$$

From equation(2) one has

$$p_{ask}(t) > p_{bid}(t). \tag{4}$$

Usually, the USD is chosen as the reference in all transactions where it is present (except for the pair GBP/USD).

The market price at time t, $p(t)$, is naturally defined as $(p_{ask}(t) + p_{bid}(t))/2$. In statistical studies of the FX market, one prefers to use the log price

$$x(t) = \frac{1}{2}\left[\log p_{ask}(t) + \log p_{bid}(t)\right] = \log \left[\frac{\lambda_{BA}}{\lambda_{AB}}\right]^{\frac{1}{2}}. \tag{5}$$

This has several advantages. Firstly, $x(t)$ only changes its sign when the reference currency changes. Secondly, x is dimensionless except for a constant and the variation of x is dimensionless. Thus, two rates as different as USD/DEM and CHF/JPY can be quantitatively compared. Finally, an increase or a decrease of the exchange rate of a given factor will result in a variation of x of the same amplitude, up to its sign.

2.2. SPREAD

The difference between the bid price and the ask price is another variable of the market state. This difference, called spread, denotes the friction $p_{ask} - p_{bid}$. Here again, one prefers to express the spread S in its logarithmic form

$$S = \log p_{ask} - \log p_{bid} = -\log \lambda_{BA}\lambda_{AB} \tag{6}$$

which has the same advantages as explained for the log price x.

From equation (2), the spread always has a positive value, which evolves as a function of the activity of the market.

2.3. CHANGE OF PRICE

Consider a trader who makes a transaction ($s_A \to s_B$) at time $t - \Delta t$ followed by the inverse one ($s_B \to s'_A$) at time t. Investing an amount s_A, he will receive

$$s'_A = s_A \frac{p_{bid}(t)}{p_{ask}(t - \Delta t)}. \tag{7}$$

His profit is $s_A[p_{bid}(t)/p_{ask}(t - \Delta t)] - s_A$. Thus, the quantity

$$\log[p_{bid}(t)/p_{ask}(t - \Delta t)] = \log p_{bid}(t) - \log p_{ask}(t - \Delta t)$$

indicates a gain or a loss, depending on whether it is positive or negative.

From this observation, the *price change* or *return*, which is a measure of the market evolution over a time interval Δt, is defined as

$$r(t, \Delta t) = x(t) - x(t - \Delta t). \tag{8}$$

Usually, for $\Delta t < 10$ minutes, the return is smaller than the spread and no profit can be expected. Note that r is symmetrically distributed and is assumed to be a stationary process.

2.4. RISK AND VOLATILITY

The risk describes the size of the fluctuations of an economical process. A large risk implies large fluctuations and a large probability of guessing wrong and losing money. The volatility $v(t, \Delta t)$ is a measure of the market fluctuations over a period T, where the price is considered with a time resolution Δt

$$v(t, \Delta t) = \frac{1}{T} \int_t^{t+T} |r(t', \Delta t)| \, dt'. \tag{9}$$

Note that $|r|$ is sometimes preferred to r^2 because it captures the behaviour of the market better [5]. This is due to the presence of fat tails when the distribution of returns is extrapolated to large values of r, making the fourth (and higher moments) of the distribution diverge. Using $|r|$ instead of the second moment puts less weight on large and uncertain values.

2.5. RETURN AND VOLATILITY PROPERTIES

In order to extract the properties of the market, it is natural to measure the probability distribution (histogram) of the return r. The main results [5] are: (i) the distribution is not Gaussian, unless $\Delta t > 2$ months; (ii) the distribution is leptokurtic (there is a more pronounced peak than for a Gaussian of the same variance); (iii) there are fat tails; (iv) the central part is well described by a Leǘy or Pareto distribution. A lot of questions are still being discussed, such as the stability under time aggregation of the distribution [7] or the description of the tails.

Studies have shown [5] that the first-order time-autocorrelation of the return $< r(t+\tau, \Delta t) r(t, \Delta t) >$ fluctuates around zero, except when the time lag τ is shorter than four minutes, in which case it is significantly negative. Thus, on average, a positive price variation is followed by a negative one, which may reflect diverging price formation strategies among the market makers. For a larger τ the price variation has no memory, but still the price is very slightly predictable in the sense that it is possible to define winning trading strategies.

It has been found [5] that the volatility also has some interesting properties. Firstly, the volatility is not uniform: clusters of high volatility are followed by clusters of low volatility. This is known as conditional heteroskedasticity and follows from the fact that an active market stimulates more transactions, whereas a quiet market acts oppositely. Secondly, the volatility autocorrelation is positive and has a long memory (3 weeks). Thirdly, daily volatility is a good predictor of hourly volatility, probably because important news first affect middle-term traders, before they propagate to short-term traders.

For what we are concerned with, the most important property of the volatility is the so-called *scaling law* [5]. It has been observed that

$$v(t, \Delta t) \sim \left(\frac{\Delta t}{\Delta T} \right)^{1/E} \tag{10}$$

for Δt ranging from 10 minutes to two months (i.e. over four decades). Here ΔT is some amplitude dependent on the FX rate. The sampling period T is several years and, for the USD/DEM market, we get

$$\frac{1}{E} = 0.582 \pm 0.006,$$

which is called the drift exponent. For a pure Gaussian random walk model, one has $1/E = 1/2$. It is also observed that $1/E$ varies from currency to currency and may change over time, although it is remarkably stable for the USD/DEM. However, the drift exponent can be smaller than $1/2$ in some cases (DEM/FRF). This may be due to anti-correlations (or anti-persistences) in the distribution of returns.

3. The Operation of the FX Market

One can distinguish two types of financial markets according to the process of prices formation: the order-driven markets and the price-driven markets. In the first type of market, for instance the stock market, the price is the result of the confrontation between the prices proposed by the buyers and the prices proposed by the sellers. The only function of the market control agencies is to verify the validity of the prices and the transactions. On the price-driven markets, on the other hand, there are a certain number of organizers, the market makers, who decide upon the prices. This is the case on the FX market, and this is largely what gives it its particular character. The traders have to follow the prices given by the market makers without being able to directly influence them, except, of course, through their collective actions.

To better understand the workings of the FX market, it is important to know the behaviour and the function of each actor on the market: the market makers, the brokers, the traders and the hedgers.

3.1. THE MARKET MAKERS AND BROKERS

As we have just said, the market makers decide on the selling and buying prices of the currencies. Naturally, they are not philanthropic companies, but they have a financial incentive. In fact, the real function of a market maker is to play the role of an intermediary between a trader who desires to exchange an amount of currency A for an amount of currency B and another trader who wants to realize the inverse operation. The profit of this operation for the market maker resides in the difference between the buying and the selling prices of a currency, i.e. the spread. Indeed, consider the case where trader 1 sells an amount s_B of currency B to the bank (for which he receives an amount s'_A of currency A) and, at the same time, trader 2 buys this same amount s_B, paying the market maker the amount s_A. Then, a simple calculation shows that the market maker's profit is

$$s_A - s'_A = (p_{ask} - p_{bid})s_B > 0. \tag{11}$$

The only risk for the market maker is the time elapsed between the two transactions. If this is too long, there is a chance that the variation of the exchange rate will screen out the effect of the spread and that the market maker will lose money. To reduce this risk, the market maker modifies the exchange rates to favour one currency to another, if there is too much demand for the second one. Evolution of the price and modifications of the spread are two means for the market makers to drive the market in some preferred direction and optimize their profit.

The role of market makers is performed by a small number (less than 100) of large banks. Of these, only about twenty or so really set the trends, the others just following in their wake.

Brokers are similar to market makers except that they do not decide upon prices. They just act as intermediaries in the market, like smaller banks or provincial branches of large banks. With the advent of fully computerized transactions, their numbers and significance are decreasing.

3.2. THE TRADERS

A trader is anyone willing to exchange one currency for another. There are three main reasons for such a transaction: (i) the need to obtain a given foreign currency for some investment; (ii) pure speculation by which a person expects to sell at a higher price than he bought at some earlier time; (iii) hedging, by which a trader protects himself against the risk of exchange rate variation. Hedgers prefer to execute their operations with the current exchange rate rather than with the unknown future rate in effect at the time of the execution of a contract.

The hedgers now represent a very important part of the transactions, but their influence is mainly noticeable on the futures market and less so on the spot market, which is the subject of our microsimulation. Indeed they deal mainly contracts on future and options. The spot market is the market where transactions are executed at the current exchange rate.

In order for a trader to make a profit, it is important to forecast the future evolution of the exchange rate. The trader's behaviour is determined by his working time horizon. The duration of a transaction is defined as the time elapsed between the purchase and sale of a given currency. Short-term traders perform intra-day transactions only, with purely speculative intentions. Middle-term traders operate over a period of a few days to two weeks. Finally, long-term traders may keep their position open for a period of a few months to one year. Since these different types of traders may have different strategies, the above three time scales may play a role in the global behaviour of the market.

There are basically two methods for predicting the future market price: (i) The "fundamentalists" use macroeconomic factors like the interest rate, inflation and unemployment to make predictions about the evolution of the market; (ii) The "chartists" rely on a technical analysis of past price history to determine the market trend. They may use more or less sophisticated mathematical tools for this purpose.

4. The Microsimulation

Modelling the behaviour of a financial system is an important issue. Stochastic processes like ARCH or GARCH (Generalized Auto-Regressive Conditional Heteroskedasticity)[8] have been considered for this purpose and can account for some properties of the price time series. Among the limitations of this approach is that there is no clear connections between the parameters and the structure of the market and the GARCH mathematical expressions.

Here we propose a model that implements the rules of the FX market we described above at the level of the market makers and traders. Such approaches have been attempted before for simulating the stock market [7, 9, 10, 1], but, to our knowledge, no such model exists for the FX market. Moreover, these approaches have always used optimal traders and have never taken into account a number of realistic factors such as constraints on the time horizon, aversion to risk, limited memory and so on. These constraints influence the expectations of the market participants, and in our opinion this diversity of behaviour is certainly at the core of the behaviour of the market.

Although we restrict ourselves to a simple situation, we will try to make the model as realistic as possible. We consider only a part of the FX market, the spot market of a unique pair of currencies. On the spot market, all the transactions are realised immediately. This excludes contracts on futures and options used mainly by hedgers. This market represents one half of the whole FX market. Limiting our model to a single pair of currencies does not seem to be a grave over-simplification. It only restricts the possibility for some traders to realise arbitrage operations between major rates and cross-rates. As these operations are quite rare, they probably do not influence the market.

We introduce two types of agents in our simulation: the market makers and the traders. The number of agents in the simulation is a parameter, typically of the order of twenty for the market makers and one thousand for the traders. Each actor has a deterministic, non-evolving behaviour. The number of strategies available for the actors is limited, but each individual has a different profile, that is a different set of parameters (time horizon, risk profile and so on) for interacting on the market.

The simulation is based on an iterative process. Each iteration corresponds to a time step composed of two main parts: (i) the price formation, (ii) the transactions. At this phase of our project, we use an homogeneous time scale, i.e. we suppress the variation of activity in the market due to its seasonal and geographical components. Our time scale is not equivalent to real time, but probably more or less similar to the rescaled time presented in some papers [11, 12] in order to have a stationary process. This has the advantage to making it easy to compare the statistical analysis of our simulation with results published in the literature. In the different runs of our simulation, we assume that the time step is one minute, but nothing prevents us from using some other value.

4.1. PRICE FORMATION

In the FX market the market makers are the actors who determine the exchange rates. Therefore, their behaviour is responsible for the process of price formation. At every time step, each market maker has a certain probability to change the rates he proposed. In this case, these new prices are communicated to all the traders. This corresponds in reality to Reuters or Telerate communicating the last rates announced by the large banks via terminals all around the world. The time series of prices used in the statistical analyses of the real market is produced from this data. Olsen & Associates has been collecting them for approximately ten years in a large database of several millions of quotations. At each time step in the simulation, a new market price is calculated from the average of all the new prices announced during that interval.

When changing the price, the market maker will follow certain rules. The first one is to minimize the time between two inverse transactions asked by different traders. In this way he can profit by the spread between the buying and the selling price. This is equivalent to continuously trying to balance demand and supply. To realize this, each market maker updates two variables, namely stock of currency A, S_A, and stock of currency B, S_B, which are the evaluations of his transactions since the beginning of the simulation. If $\Delta = S_A - p_{bid}S_B > 0$, then the demand for currency B is too great. To correct this, the market maker will raise the prices. Inversely, if $\Delta < 0$, the market maker can lower his prices to make currency B more attractive. The correction of the price will be proportional to the value of Δ, but with two limitations. First of all it is important that the market maker does not propose prices too distant from the market prices. Keeping a relatively neutral position insures that the market maker will have a sufficiently large number of transactions. Consequently, if the old price is already higher than the current market price, a correction to the rise will be moderated and a price drop will be increased according to the direction taken by the market maker. The second limitation is a bound preventing the correction to be higher than a fraction of the spread. Indeed, a large variation increases the risk for the market maker and prevents him from taking advantage of the spread.

To reduce the risk, the market maker can also play with the spread and modify it following his needs. In general, the spread takes fixed values such as 5, 10, 15, ... points, where a point is the unit of change of prices. For example, in the USD/DEM rate, a point is equal to 10^{-4}. Usually the market makers use a spread of 5 points, but when a market maker proposes a price very far from the market price, he can increase the spread. In our simulation, each market maker modifies his spread such that p_{bid} is smaller than p_{market} and that p_{ask} is greater then p_{market}, where p_{market} is the average bid and ask price, calculated for the whole market at time $t-1$.

These rules can be expressed in equations. Let $p_i(t)$ be the average of the buying and the selling prices of market maker i at time t, and let $\delta(t)$ be the spread. The price correction $\Delta p_i(t)$ is given by

$$\begin{aligned}
\Delta p_i(t) &= \begin{cases} \max(c_1\Delta - (p_i(t-1) - p_{\text{market}}(t-1)), -c_2\delta(t-1)), & \text{if } \Delta < 0 \\ \min(c_1\Delta - (p_i(t-1) - p_{\text{market}}(t-1)), c_2\delta(t-1)), & \text{if } \Delta > 0 \end{cases} \\
\delta(t) &= \text{int}\left[\frac{|p(t) - p_{\text{market}}(t-1)|}{2.5\text{points}}\right] 5\text{points} \\
p_{ask}(t) &= p_i(t) + \delta(t)/2 \\
p_{bid}(t) &= p_i(t) - \delta(t)/2.
\end{aligned}$$

The quantities c_1 and c_2 are constants that specify the market maker profile.

4.2. ANALYSIS AND TRANSACTIONS

The process of transactions contains two stages. The first one is an analysis of the market evolution by the traders with the purpose of predicting a tendency and deciding on whether to make a deal. The second one consists of the interaction between a trader and a market maker which will define the price and the amount of the transaction.

In the present state of our simulation the traders only use the technical analysis, i.e. a study of the time series of past prices as a means of prediting the market. Currently, only one strategy is available for the traders. It is based on the use of the moving averages and the momentum. This strategy is used to open a position, i.e. to decide when and in which direction (buy or sell) a trader should enter the market. To close their position, our traders use another rule based on two bounds, the maximum tolerated loss and the expected profit. Six parameters will allow us to create a personal profile for each trader. These different profiles will be responsible for the differences observed in the positions taken by the traders.

The first parameter T is the time horizon on which each trader deals. T can belong to one of three ranges of values corresponding to the different types of traders: (i) less than one day for the intra-day traders, (ii) from 8 to 15 days for the middle-term traders and (iii) from 30 to 90 days for the long-term traders. This time horizon is a constraint on the maximum length of time a position can be open. This parameter also influences the values of the other parameters by fixing their orders of magnitude.

Three parameters are related to the use of the momentum strategy. The first one, T_m, represents the memory of the trader and is used to calculate the moving average $\text{ema}_{T_m}(t)$ of the price series. The moving average is a weighted average of a time series, where the weight decreases according to how old the data is. In the present case, we will take an exponential decrease [13].

$$\mathrm{ema}_{T_m}(t) \equiv \frac{1}{T_m + 1} \sum_{i=-\infty}^{t} \left(\frac{T_m}{T_m + 1} \right)^{t-i} x(i) \tag{12}$$

The momentum $m(t) \equiv \mathrm{ema}_{T_1}(t) - \mathrm{ema}_{T_2}(t)$ is the difference between two moving averages calculated for two different time scales T_1 and T_2. T_1 is always much smaller than T_2. Usually one chooses T_1 equal to zero, i.e. the moving average is equivalent to the last value. Thus the momentum takes the value

$$m(t) \equiv x(t) - \mathrm{ema}_{T_m}(t). \tag{13}$$

The momentum is an indicator of the market trend, because it gives the distance between the present price and its average. In our model we introduce two parameters ϵ_1 and ϵ_2 ($\epsilon_1 < \epsilon_2$). They are thresholds which will drive the trader to take a decision according to the value of the momentum. When the momentum exceeds the first threshold ϵ_1, the price is on the rise and therefore the trader will buy currency B. If the momentum exceeds the second threshold ϵ_2, the trader supposes that the rise is so big that it will very soon pass a critical point, which will lead to a period of fall. In order to be ahead of the market, he will bear the market and buy currency A. The thresholds $-\epsilon_1$ and $-\epsilon_2$ are used similarly when the momentum is negative and decreasing, but with the inverse action for traders. In the range $[-\epsilon_1, \epsilon_1]$, the momentum does not indicate a clear tendency and the traders do not open any positions.

The last two parameters of the traders define their risk profile. r_{min} and r_{max} ($r_{min} < 0 < r_{max}$) are two bounds used to close an open position. If an open position can produce a return larger than r_{max}, the trader will close the position. The transaction has generated the expected profit. On the contrary, if the return is lower than r_{min}, the trader has to close his position, because he has reached the maximum tolerated loss.

After the analysis of the market evolution, the traders contact the market makers to realise the transactions. Usually, a trader is related to only one or a few market makers, where he has a deposit of at least 10% of the desired operation. The traders choose the best exchange rate between these market makers. In our simulation a market maker never refuses a transaction, similarly to the real market.

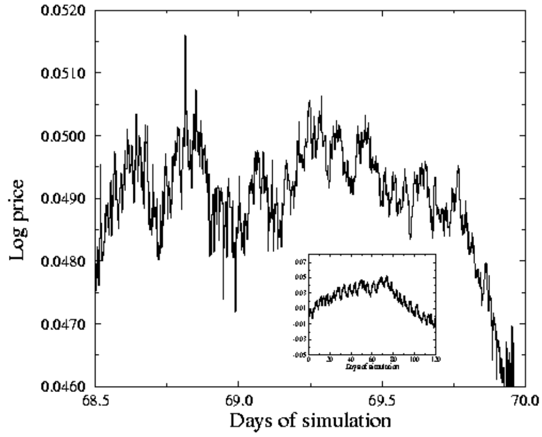

Figure 1. Evolution of the market over a few days. The small graph represents the evolution on a larger time scale (4 months) and shows regular and periodic patterns

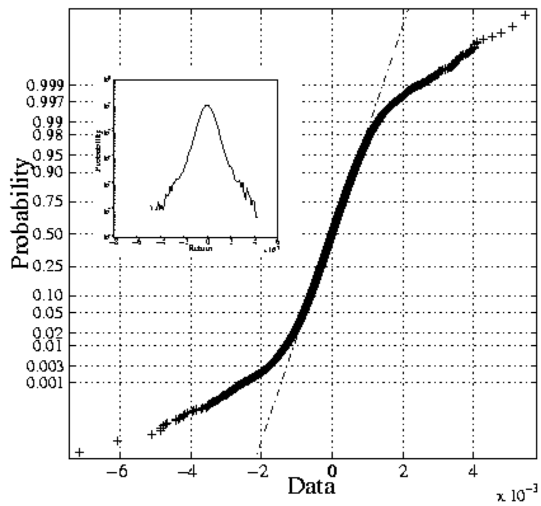

Figure 2. Cumulative frequency distribution of the return (Δt = 10 minutes) on a Gaussian scale. A Gaussian distribution would be represented by a straight line. The small graph shows the probability distribution of the return.

5. Results

This work is still in progress. Thus we can only present preliminary results. We have implemented our simulation on a parallel computer (IBM SP2) to enable us to execute simulations with a large number of agents on a time scale of many years in a reasonable time. In Fig. 1, we show the evolution of

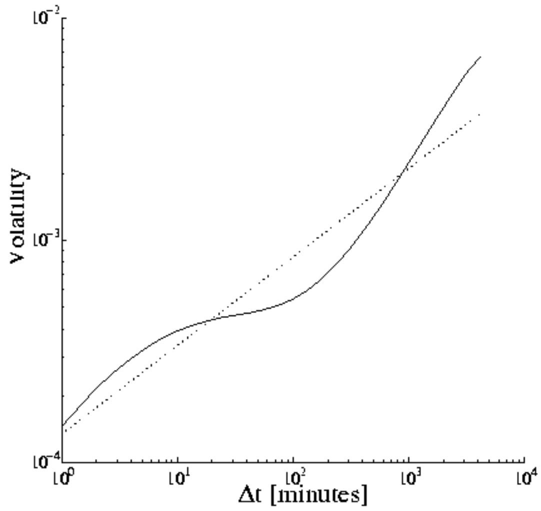

Figure 3. Scaling law for the volatility. The dashed line represents the volatility of a Gaussian random walk (slope = 0.5).

an artificial market of 24 market makers and 1024 traders. On a short time scale the evolution seems rather chaotic and the amplitude of the variations is larger than expected in a Gaussian random walk. Fig. 2 confirms this fact by showing the presence of fat tails in the short-term return distribution. The departure from the straight line of the distribution in a Gaussian plot implies that extreme events are more frequent than in a random walk.

On a large time scale (Fig. 1) the results are not so good, because we observe regular and periodic patterns. This implies that the price variations are bounded. We will have to improve our model to suppress this undesirable effect. The reason for these patterns is perhaps that the strategy of the market makers is too basic, or that the behaviour of the midddle-term and long-term traders is too similar.

This suspect periodicity in the evolution of the market prevents us from observing the scaling law of the volatility (see Fig. 3). It seems that there are two regimes in the graph, one on the very short time scale and one on the middle and long time scales. Each part could be represented by a scaling law with a drift exponent higher than 0.5. Both exponents are in agreement with the findings of [14, 15].

Fig. 4 shows the autocorrelation $< r(t+\tau, \Delta t) r(t, \Delta t) >$ of the short-term return for different time lags τ. As expected, we observe a significantly negative autocorrelation for a τ smaller than a few minutes. After 10 minutes, the autocorrelation cannot be distinguished from the autocorrelation of a Gaussian random walk process.

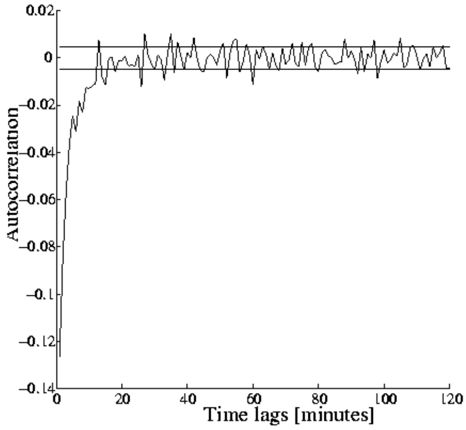

Figure 4. First-order autocorrelation of the returns for different time lags in minutes ($\Delta t = 1$ minute). The two horizontal lines represent the 95% confidence interval of a Gaussian random process.

6. Conclusion

These preliminary results are promising, because our model captures many of the qualitative features of the real market. It seems that our model is very efficient on the short-term scale, but encounters more problems on larger time scales. To solve this difficulty, we will add new types of strategies for the traders, based on different indicators such as the volatility index or the stochastic indicators. In this way we hope to increase the probability of traders doing inverse operations at the same time, and thus breaking the periodic behaviour.

Acknowledgments

The authors wish to thank people from Olsen and Associates and specially O. Pictet for helpful discussions on the workings of the FX market.

References

1. P. Bak, M. Paczuski, and M. Shubik. Price variations in a stock market with many agents. *Physica A*, 1997.
2. Moshe Levy, Haim Levy, and Sorin Solomon. Microscopic simulation of the stock market: the effect of microscopic diversity. *Journal de Physique I*, 5:1087–1107, 1995.
3. Aki-Hiro Sato and Hideki Takayasu. Dynamical numerical models of stock market price: from microscopic determinism to macroscopic randomness. submitted to Physica A, 1997.
4. G. Caldarelli, M. Marsili, and Y.-C. Zhang. A prototype model of stock exchange. Preprint, 1997.

5. Dominique M.Guillaume, Michel M. Dacorogna, Rakhal R. Dav, Ulrich A. Mller, Richard B. Olsen, and Olivier V.Pictet. From the bird's eye to the microscope: A survey of new stylized facts of the intra-daily foreign exchange markets. *Finance and Stochastics*, 1:95–129, 1997.
6. Michel M. Dacorogna, Cindy L. Gauvreau, Ulrich A. Mller, Richard B. Olsen, and Olivier V. Pictet. Changing time scale for short-term forecasting in financial markets. *Journal of Forecasting*, 15(3):203–227, 1996. special issue on non-linear forecasting.
7. Rosario N. Mantegna and H. Eugene Stanley. Scaling behaviour in the dynamics of an economic index. *Nature*, 376:46–49, July 1995.
8. Ulrich A. Mller, Michel M. Dacorogna, Rakhal R. Dav, Richard B. Olsen, Olivier V. Pictet, and Jakob E. von Weizscker. Volatilities of different time resolutions: Analysing the dynamics of market components. *Journal of Empirical Finance*, 4(2-3):213–240, June 1997.
9. W. Brian Arthur. Complexity in economics and financial markets. *Complexity*, 1:20–25, 1995.
10. Blake LeBaron. Experiments in evolutionary finance. Technical report, University of Wisconsin, 1995.
11. Michel M. Dacorogna, Ulrich A. Mller, Rakhal J. Nagler, Richard B. Olsen, and Olivier V. Pictet. A geographical model for the daily and weekly seasonal volatility in the foreign exchange market. *Journal of International Money and Finance*, 12(4):413–438, 1993.
12. S. Galluccio, G. Caldarelli, M. Marsili, and Y.-C. Zhang. Scaling in currency exchange. Preprint, 1997.
13. Ulrich A. Mller. Specially weighted moving averages with repeated application of the ema operator. Proprietary - Internal document UAM. 1991-10-14, Olsen & Associates, 1991.
14. Yanhui Liu, Pierre Cizeau, Martin Meyer, C.-K. Peng, and H. Eugene Stanley. Correlations in economic time series. submitted to Elsevier Science, 1997.
15. N. Vandewalle and M. Ausloos. Sparseness and roughness of foreign exchange rates. Preprint, 1997.