MODELING SCALING BEHAVIOR IN THE GROWTH DYNAMICS OF ORGANIZATIONS

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1. Introduction

The goal of statistical physics is to predict the macroscopic behavior of systems composed of many interacting units from microscopic interactions. The studies of the '60s and '70s on critical phenomena [1, 2, 3] and fractal geometry [4] provided an array of techniques and concepts that have proven themselves relevant for studying complex systems. A key concept emerging from the study of critical phenomena is *universality* which enables us, under certain conditions, to predict macroscopic behavior semi-quantitatively — in the form of scaling laws — without too much knowledge of the microscopic behavior. That is, classes of interactions, which differ in some details, can be described by the same scaling laws. The new tools of statistical physics have found immediate application in many fields such as biophysics, medicine, geomorphology, geology, evolution, ecology or meteorology.

Problems in economics appear particularly suited for the application of the methods of statistical physics: There, a large number of agents interact with each other giving rise to macroscopic — macroeconomic — behavior. Recognizing these similarities, several research groups have turned their attention to problems in economics [5, 6] and finance [7]–[17]. Simultaneously, the concepts of statistical physics (e.g., self-organization, scale-free phenomena) have started to penetrate the study of economics [18].

Here, we extend the study of Ref. [6] on the growth rate of manufacturing firms. One of our motivations for the present study is the considerable recent interest in economics in developing a richer theory of the firm [19]–[42]. In standard microeconomic theory, a firm is viewed as a production function for transforming inputs such as labor, capital, and materials into output [22, 29, 35]. In contrast to this static model, recent work on firm dynamics emphasizes the effect of how firms learn over time about their efficiency relative to competitors [28, 43, 44]. The production dynamics captured in these models are not, however, the only source of actual firm dynamics. Most notably, the existing models do not account for the time needed to assemble the organizational infrastructure needed to support the scale of production that typifies modern corporations.

We studied all United States (US) manufacturing publicly-traded firms from 1974 to 1993. The source of our data is Compustat which is a database on all publicly-traded firms in the US. Compustat obtains this information from reports that publicly traded firms must file with the US Securities and Exchange Commission. The database contains detailed information on each firm. Among the items included are "*sales*," "*cost of goods sold*," "*assets*," "*number of employees*," and "*property, plant, & equipment*."

Another item provided for each firm is the Standard Industrial Classification (SIC) code. In principle, two firms in the same primary SIC code are in the same market; that is, they compete with each other. In practice, defining markets is extremely difficult [45]. More important for our analysis, virtually all modern firms sell in more than one market. Firms that operate in different markets do report some disaggregated data on the different activities. For example, while Philip Morris was originally a tobacco producer, it is also a major seller of food products (since its acquisition of General Foods) and of beer (since its acquisition of Miller Beer). Philip Morris does report its sales of tobacco products, food products, and beer separately. However, firms have considerable discretion in how to report information on their different activities, and differences in their choices make it difficult to compare the data across firms.

In this paper, the only use we make of the primary SIC codes in Compustat is to restrict our attention to manufacturing firms. Specifically, we include in our sample all firms with a major SIC code from 2000–3999. We do not use the data from the individual business segments of a firm, nor do we divide up the sample according to primary SIC codes. We should acknowledge that this choice is at odds with the mainstream of economic analysis. In economics, what is commonly called the "theory of the firm" is actually a theory of a business unit. To build on the Philip Morris example, economists would likely not use a single model to predict the behavior of Philip Morris. At the very least, they would use one model for the tobacco division, one for the food division, and one for the beer division. Indeed, given the available data, they might construct a completely separate model of, say, the sales of Maxwell House coffee. Because the standard model of the firm applies to business units, it does not yield any prediction about the distribution of the size of actual, multi-divisional firms or their growth rates.

On the other hand, the approach we take in this study is part of a distinguished tradition. First, there is a large body of work by economics Nobel laureate H. Simon [30] and various co-authors that explored the stochastic properties of the dynamics of firm growth. Also, in a widely cited article (that nonetheless has not had much impact on mainstream economic analysis), R. Lucas, also a Nobel laureate, suggests that the distribution of firm size depends on the distribution of managerial ability in the economy rather than on the factors that determine size in the conventional theory of the firm [31].

n summary, the first goal of our study is to uncover empirical scaling regularities about the growth of firms that could serve as a test of models for the growth of firms. We find: (*i*) the distribution of the logarithm of the growth rates for firms with approximately the same size displays an exponential form, and (*ii*) the fluctuations in the growth rates — measured by the standard deviation of this distribution — scale as a power law with firm size. The second goal of our study is to develop a "microscopic model," based on reasonable assumptions, that explains the observed empirical results.

The paper is organized as follows: In Sect. 2, we review the economics literature on the growth of firms. In Sects. 3 and 4, we present our empirical results for publicly-traded US manufacturing firms. In Sect. 5, we discuss the relevance of our empirical findings. In Sect. 6, we present a model that can account for all empirical results. Finally, in Sect. 7, we present some concluding remarks.

2. Background

In 1931, the French economist Gibrat proposed a simple model to explain the empirically observed size distribution of firms [20]. He made the following assumptions: (*i*) the growth rate R of a firm is independent of its size (this assumption is usually referred to by economists as the *law of proportionate effect*), (*ii*) the successive growth rates of a firm are uncorrelated in time, and (*iii*) the firms do not interact.

In mathematical form, Gibrat's model is expressed by the stochastic process:

$$S_{t+\Delta t} = S_t(1 + \epsilon_t), \tag{1}$$

where $S_{t+\Delta t}$ and S_t are, respectively, the size of the firm at times $(t + \Delta t)$ and t, and ϵ_t is an uncorrelated random number with some bounded distribution, usually assumed to be Gaussian, and variance much smaller than one. Hence $\log S_t$ follows a simple random walk and, for sufficiently large time intervals $u \gg \Delta t$, the growth rates

$$R_u \equiv \frac{S_{t+u}}{S_t} \tag{2}$$

are log-normally distributed. If we assume that all firms are born at approximately the same time and have approximately the same initial size, then the distribution of firm sizes is also log-normal. This prediction from the Gibrat model is approximately correct [46, 47].

There is, however, considerable evidence that contradicts Gibrat's underlying assumptions. The most striking deviation is that the fluctuations of the growth rate measured by the relative standard deviation $\sigma_1(S)$ decline with an increase in firm size. This was first observed by Singh and Whittington [48] and confirmed by others [6], [49]–[53]. The negative relationship between growth fluctuations and size is not surprising because large firms are likely to be more diversified. Singh and Whittington state that the decline of the standard deviation with size is not as rapid as if the firms consisted of independently operating subsidiary divisions. The latter would imply that the relative standard deviation decays as $\sigma_1(S) \sim S^{-1/2}$ [48].

The situation for the mean growth rate is less clear. Singh and Whittington [48] consider the assets of firms and observe that the mean growth rate increases slightly with size. However, the work of Evans [49] and Hall [50], using the number of employees to define the firm's size, suggests that the mean growth rate declines slightly with size. Dunne et al. [51] emphasize the effect of the failure rate of firms and the effect of the ownership status (single- or multi-unit firms) on the relation between size and mean growth rate. They conclude that the mean growth rate is always negatively related with size for single-unit firms; but for multi-unit firms, the growth rate increases modestly with size because the reduction in their failure rates overwhelms a reduction in the growth of non-failing firms [51].

Another testable implication of Gibrat's law is that the growth rate of a firm is uncorrelated in time. However, the empirical results in the literature are not conclusive. Singh and Whittington [48] observe positive first order correlations in the 1-year growth rate of a firm (persistence of growth) whereas Hall [50] finds no such correlations. The possibility of negative correlations (regression towards the mean) has also been suggested [54, 55].

3. Size distribution of publicly-traded firms

In the following sections, we study the distribution of firm sizes and growth rates. To do so, one problem that must be confronted is the definition of firm size. Measures generally used by economists to define size are "*sales*," "*number of employees*," "*cost of goods sold*," "*property, plant & equipment*," and "*assets*." As we discuss below, we obtain similar results for all of these measures. We begin by describing the growth rate of sales. To make the values of sales in different years comparable, we adjust all values to 1987 dollars by the GNP price deflator.

In the limit of small annual changes in S, we can define the relative growth rate as

$$r_1 \equiv \ln R_1 = \ln \frac{S_1}{S_0} \approx \frac{S_1 - S_0}{S_0}\,, \tag{3}$$

where S_0 is the size of a firm in a given year and S_1 its size the following year.

Stanley *et al.* determined the size distribution of publicly-traded manufacturing firms in the US [46]. They found that for 1993, the data fit to a good degree of approximation a log-normal distribution. These results have been recently confirmed by Hart and Oulton [47] for a sample of approximately 80,000 United Kingdom firms. Here, we present a study of the distribution for a period of 20 years (from 1974 to 1993).

Figure 1a shows the total number of publicly-traded manufacturing firms present in the database each year. Figure 1b shows the distribution of firm size in each year from 1974–1993. Particularly above the lower tails, the distributions lie virtually on top of each other. Thus the distribution is stable over this period. This is surprising because there is no existing theoretical reason to expect that the size distribution of firms could remain stable. Further, this result contradicts the predictions of the Gibrat model. Equation (1) implies that the distribution of sizes of firms should get broader with time. In fact, the variance of the distribution should increase linearly in time. Thus, we must conclude that other factors, not included in Gibrat's assumptions, must have important roles.

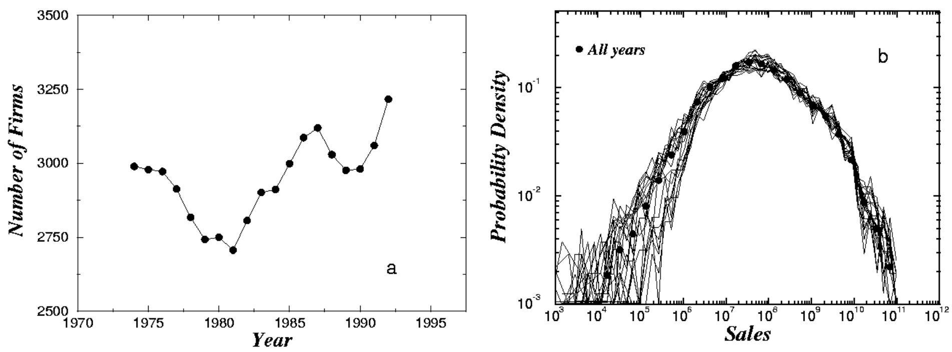

Figure 1. (a) Number of publicly-traded manufacturing firms in the US for the period 1974–1993. (b) Probability density of the logarithm of the sales for publicly-traded manufacturing firms (with standard industrial classification index of 2000-3999) in the US for *each* of the years in the 1974–1993 period. All the values for sales were adjusted to 1987 dollars by the GNP price deflator. Also shown (solid circles) is the average over the 20 years. Note that the distribution is approximately stable over the period.

One obvious factor not captured by the Gibrat assumption is the entry of new firms. We find that the size distribution of new publicly-traded firms is approximately a log-normal with an average slightly smaller than for existing firms [56]. We would expect new firms to be much smaller on average than existing ones. However, new firms can come about through the merger of two existing firms, in which case the new firm is bigger than either of the pre-existing firms. Another way that new firms come into existence is that very large firms divest themselves of divisions that are, by themselves, large businesses. An example is AT&T's recent divestiture of its manufacturing division (Lucent) and its computer division (NCR).

Another factor not included in Gibrat's assumptions is the "dying" of firms. We find that this distribution is quite similar to the distribution for all firms [56]. Thus, it suggests that the probability for a firm to leave the market, whether by merger, change of name, or bankruptcy, is nearly independent of size [56].

4. The distribution of growth rates

The distribution $p(r_1|s_0)$ of the growth rates from 1974 to 1993 is shown in Fig. 2a for three different values of the initial sales. Remarkably, these curves can be approximated by a simple "tent-shaped" form. Hence the distribution is not Gaussian — as expected from the Gibrat approach [20]

— but rather is exponential [6],

$$p(r_1|s_0) = \frac{1}{\sqrt{2}\sigma_1(s_0)} \exp\left(-\frac{\sqrt{2}\,|r_1 - \bar{r}_1(s_0)|}{\sigma_1(s_0)}\right). \tag{4}$$

The straight lines shown in Fig. 2a are calculated from the average growth rate $\bar{r}_1(s_0)$ and the standard deviation $\sigma_1(s_0)$ obtained by fitting the data to Eq. (4). The tails of the distribution in Fig. 2a are somewhat fatter than Eq. (4) predicts. This deviation is the opposite of what one would find if the distribution were Gaussian. We find that the data for *each* annual interval from 1974–1993 also fit well to Eq. (4), with only small variations in the parameters $\bar{r}_1(s_0)$ and $\sigma_1(s_0)$.

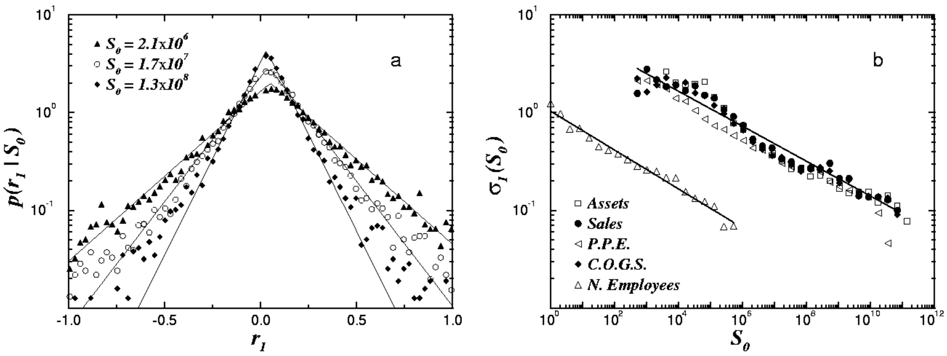

Figure 2. (a) Probability density $p(r_1|s_0)$ of the growth rate $r \equiv \ln(S_1/S_0)$ for all publicly-traded US manufacturing firms in the 1994 Compustat database with Standard Industrial Classification index of 2000–3999. The distribution represents all annual growth rates observed in the 19-year period 1974–1993. We show the data for three different bins of initial sales (with sizes increasing by powers of 8): $8^7 < S_0 < 8^8$, $8^8 < S_0 < 8^9$, and $8^9 < S_0 < 8^{10}$. Within each sales bin, each firm has a different value of R, so the abscissa value is obtained by binning these R values. The solid lines are exponential fits to the empirical data close to the peak. We can see that the wings are somewhat "fatter" than is predicted by an exponential dependence. (b) Standard deviation of the 1-year growth rates for different definitions of the size of a firm as a function of the initial values. Least squares power law fits were made for all quantities leading to the estimates of β: 0.18 ± 0.03 for "assets," 0.20 ± 0.03 for "sales," 0.18 ± 0.03 for "number of employees," 0.18 ± 0.03 for "cost of goods sold," and 0.20 ± 0.03 for "plant, property & equipment." The straight lines are guides for the eye and have slopes 0.19.

4.1. STANDARD DEVIATION OF THE GROWTH RATE

Next, we study the dependence of $\sigma_1(s_0)$ on s_0. As is apparent from Fig. 2, the width of the distribution of growth rates decreases with increasing S_0. We find that $\sigma_1(S_0)$ is well approximated for 8 orders of magnitude (from sales of 10^3 dollars up to sales of 10^{11} dollars) by the law [6]

$$\sigma_1(S_0) \sim {S_0}^{-\beta}\,, \tag{5}$$

where $\beta = 0.20 \pm 0.03$. Figure 2b displays σ_1 vs. S_0, and we can see that Eq. (5) is indeed verified by the data.

4.2. OTHER MEASURES OF SIZE

In order to test further the robustness of our findings, we perform a parallel analysis for the number of employees. We find that the analogs of $p(r_1|s_0)$ and $\sigma_1(s_0)$ behave similarly. For example, Fig. 2b shows the standard deviation of the number of employees, and we see that the data are linear over roughly 5 orders of magnitude, from firms with less than 10 employees to firms with almost 10^6 employees. The slope $\beta = 0.18 \pm 0.03$ is the same, within the error bars, as found for the sales.

We find that Eqs. (4) and (5) approximately describe three additional indicators of a firm's size, (*i*) assets (with exponent $\beta = 0.18 \pm 0.03$) (*ii*) cost of goods sold ($\beta = 0.18 \pm 0.03$) and (*iii*) property, plant & equipment ($\beta = 0.20 \pm 0.03$).

5. Discussion of empirical results

What is remarkable about Eqs. (4) and (5) is that they approximate the growth rates of a diverse set of firms. They range not only in their size but also in what they manufacture. The conventional economic theory of the firm is based on production technology, which varies from product to product. Conventional theory does not suggest that the processes governing the growth rate of car firms should be the same as those governing, e.g., pharmaceutical or paper firms.

Indeed, our findings are reminiscent of the concept of universality found in statistical physics, where different systems can be characterized by the same fundamental laws, independent of "microscopic" details. Thus, we can pose the question of the universality of our results: Is the measured value of the exponent β due to some averaging over the different industries, or is it due to a universal behavior valid across all industries? As a "robustness check," we split the entire sample into two distinct intervals of SIC codes. It is visually apparent in Fig. 3a that the same behavior holds for the different samples of industries.

In statistical physics, scaling phenomena of the sort that we have uncovered in the sales and employee distribution functions are sometimes represented graphically by plotting a suitably "scaled" dependent variable as a function of a suitably "scaled" independent variable. If scaling holds,

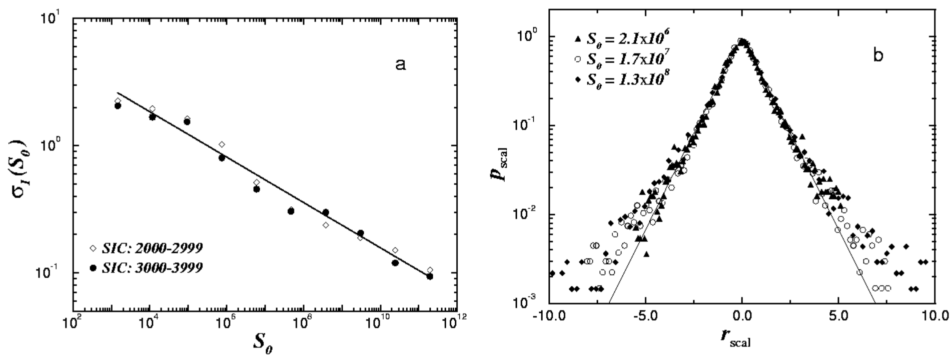

Figure 3. (a) Dependence of σ_1 on S_0 for two subsets of the data corresponding to different values of the SIC codes. In principle, firms in different subsets operate in different markets. The figure suggests that our results are universal across markets. (b) Scaled probability density $p_{\text{scal}} \equiv \sqrt{2}\sigma_1(s_0)p(r_1|s_0)$ as a function of the scaled growth rate $r_{\text{scal}} \equiv \sqrt{2}[r_1 - \bar{r}_1(s_0)]/\sigma_1(s_0)$. The values were rescaled using the measured values of $\bar{r}_1(\text{s}_0)$ and $\sigma_1(s_0)$. All the data collapse upon the universal curve $p_{\text{scal}} = \exp(-|r_{\text{scal}}|)$ as predicted by Eqs. (4) and (5).

then the data for a wide range of parameter values are said to "collapse" upon a *single* curve. To test the present data for such data collapse, we plot in Fig. 3b the scaled probability density $p_{\text{scal}} \equiv \sqrt{2}\sigma(s_0)p(r_1|\text{s}_0)$ as a function of the scaled growth rates of both sales and employees $r_{\text{scal}} \equiv \sqrt{2}[r_1 - \bar{r}_1(s_0)]/\sigma(s_0)$. The data collapse relatively well upon the single curve $p_{\text{scal}} = \exp(-|r_{\text{scal}}|)$. Our results for (*i*) cost of goods sold, (*ii*) assets, and (*iii*) property, plant & equipment are equally consistent with such scaling.

For "physical" phenomena, power law scaling is usually associated with critical behavior — thus, requiring a particular set of parameter values — or with scale-free nonequilibrium processes [57, 58]. For example, in the Ising model there is a particular value of the strength of the interaction between the units composing the system that generates correlations extending throughout the entire system and leads to power law distributions. Although self-organized criticality [59] has been the preferred explanation for these results, it is difficult to imagine that for all these diverse systems, the parameters controlling the dynamics spontaneously self-tune to their critical values.

In the next section, we discuss an alternative mechanism, in the spirit of scale-free growth processes, that could explain how power law scaling in biological or social sciences can emerge even in the absence of critical dynamics. The guiding principles for our approach are: (*i*) the units composing the system have a complex evolving structure (e.g., the firms competing in an economy are composed of divisions, the cities in a country competing for the mobile population are composed of distinct neighborhoods, the population of some species living in a given ecosystem might be composed of groups living in different areas), and (*ii*) the size of the subunits composing each unit evolves according to a random multiplicative process.

6. Modeling the growth of firms

In this section, we develop a model [60] that dynamically builds a diversified, multi-divisional structure, reproducing the fact that a typical firm passes through a series of changes in organization, growing from a single-product, single-plant firm, to a multidivisional, multiproduct firm [27]. The model reproduces a number of empirical observations for a wide range of values of parameters and provides a possible explanation for the robustness of the empirical results. Due to our encouraging results for the case of firm growth, our model may offer a generic approach to explain power law distributions in other complex systems [61].

The model, illustrated in Fig. 4, is defined as follows. A firm is created with a single division, which has a size $\xi_1(t = 0)$. The size of a firm $S \equiv \Sigma_i \xi_i(t)$ at time t is the sum of the sizes of the divisions $\xi_i(t)$ comprising the firm. We define a minimum size $S_{\min}$ below which a firm would not be economically viable, due to the competition between firms; $S_{\min}$ is a characteristic of the industry in which the firm operates. We assume that the size of each division i of the firm evolves according to a random multiplicative process. We define

$$\Delta\xi_i(t) \equiv \xi_i(t)\ \eta_i(t)\,, \tag{6}$$

where $\eta_i(t)$ is a Gaussian-distributed random variable with zero mean and standard deviation V independent of ξ_i. The divisions evolve as follows:

(*i*) If $\Delta\xi_i(t) < S_{\min}$, division i evolves by changing its size, and $\xi_i(t+1) = \xi_i(t) + \Delta\xi_i(t)$. If its size becomes smaller than $S_{\min}$ — i.e. if $\xi_i(t+1) < S_{\min}$ — then with probability p_a, division i is "absorbed" by division 1. Thus, the parameter p_a reflects the fact that when a division becomes very small it will no longer be viable due to the competition between firms.

(*ii*) If $\Delta\xi_i(t) > S_{\min}$, then with probability $(1 - p_f)$, we set $\xi_i(t+1) = \xi_i(t) + \Delta\xi_i(t)$. With a probability p_f, division i does not change its size — so that $\xi_i(t+1) = \xi_i(t)$ — and an altogether new division j is created with size $\xi_j(t+1) = \Delta\xi_i(t)$. Thus, the parameter p_f reflects the tendency to diversify: the larger is p_f, the more likely it is that new divisions are created.

The dynamics are thus controlled by three independent parameters: V, p_a, and p_f — $S_{\min}$ just sets the scale, so the results of the model do not

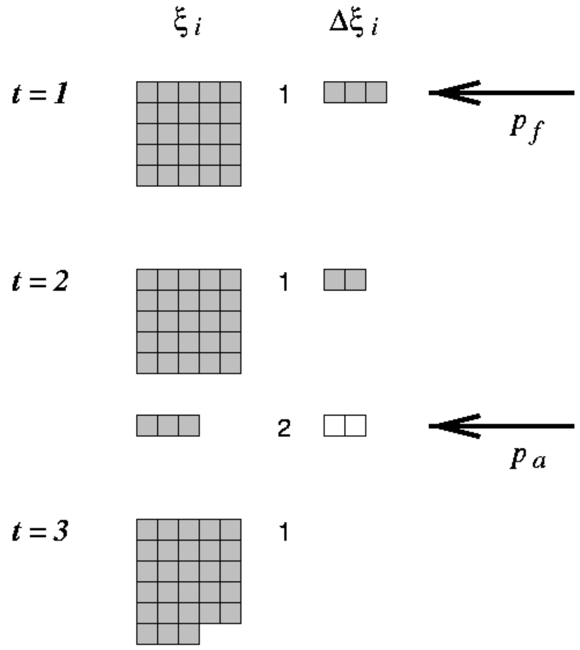

Figure 4. Schematic representation of the time evolution of the size and structure of a firm. We choose $S_{\min} = 2$, and $p_f = p_a = 1.0$. The first column of full squares represents the size ξ_i of each division, and the second column represents the corresponding change in size $\Delta\xi_i$. Empty squares represent negative growth and full squares positive growth. We assume, for this example, that the firm has initially one division of size $\xi_1 = 25$, represented by a 5×5 square. At $t = 1$, division 1 grows by $\Delta\xi_1 = 3$. A new division, numbered 2, is created because $\Delta\xi_1 > S_{\min} = 2$, and the size of division 1 remains unchanged, so for $t = 2$, the firm has 2 divisions with sizes $\xi_1 = 25$ and $\xi_2 = 3$. Next, divisions ξ_1 and ξ_2 grow by 2 and -2, respectively. Division 2 is absorbed by division 1, since otherwise its size would become $\xi_2 = 3 - 2 = 1$ which is smaller than $S_{\min}$. Thus, at time $t = 3$, the firm has only one division with size $\xi_1 = 25 + 2 + 1 = 28$. Note that if division 1 were absorbed, then division 2 would absorb division 1 and would then be renumbered 1. If division 1 is absorbed and there are no more divisions left, the firm "dies."

depend on its value. We assume that there is a broad distribution of values of $S_{\min}$ in the system because firms in different activities will have different constraints.

In Fig. 5, we compare the predictions of the model for the distribution of firm sizes in the stationary state with the empirical data [6]. The stationary state is reached after approximately ten "years", provided that new firms are created regularly. We define one "year" as ℓ iterations of our rules applied to each firm, and we find no significant dependence of the results on the value of ℓ for $\ell > 10$. We find similar results for a wide range of

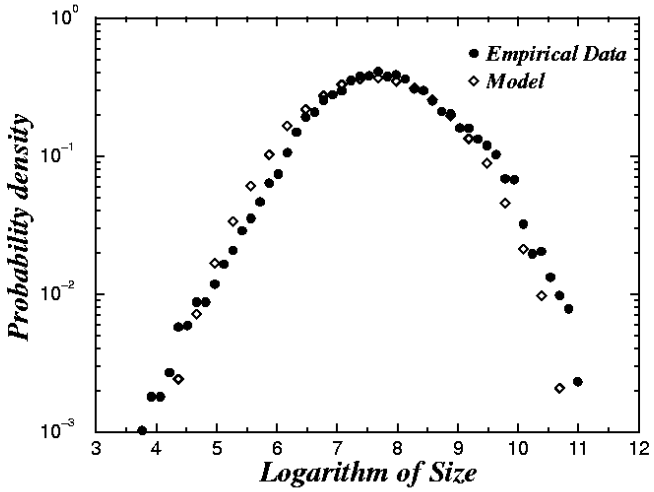

Figure 5. Probability density of the logarithm of firm size for the model and for US publicly-traded manufacturing firms in the 1994 "Compustat" data base. These results were obtained drawing $\log S_{\min}$ from a Gaussian distribution with average value $\log(5 \times 10^5)$ and width $\mathcal{D} = 5$. Similar results would be obtained for other broad distributions of $S_{\min}$. The numerical simulations were performed with parameters $V = 0.15$, $p_f = 0.8$, $p_a = 0.05$, and $\ell = 50$ (for these parameter values, the actual probability of a new division being created per division and per iteration is approximately 0.01).

parameters: $V = 0.1 - 0.2$, $p_a = 0.01 - 1$, and $p_f = 0.1 - 1.0$.

We find that $p(r_1|S)$ is quite similar in form to the empirical results [6]. Figure 6b compares $\sigma_1(S)$ with the empirical data of Ref. [6]: for both, Eq. (5) holds with $\beta = 0.17 \pm 0.03$. Equations (4)–(5) allow us to scale the growth rate distributions for different firm sizes [Fig. 6c].

6.1. STRUCTURE OF THE FIRM

We next address the question of the structure of a given firm. To this end, we study the dependence of average size of the subunits $\bar{\xi}_i$ on firm size for the model. We find [60]

$$\bar{\xi}_i \sim S^{\alpha} , \tag{7}$$

with $\alpha = 0.66 \pm 0.05$. Next, we study the dependence of the average number of subunits of a firm $\bar{N}$ on firm size. We find [60]

$$\bar{N} \sim S^{1-\alpha} , \tag{8}$$

with the same value of the scaling exponent α as above.

The results described by Eqs. (7)-(8) are in qualitative agreement with empirical studies [32] that show larger firms to be more diversified. Moreover, since N does not change much during a year and assuming that the

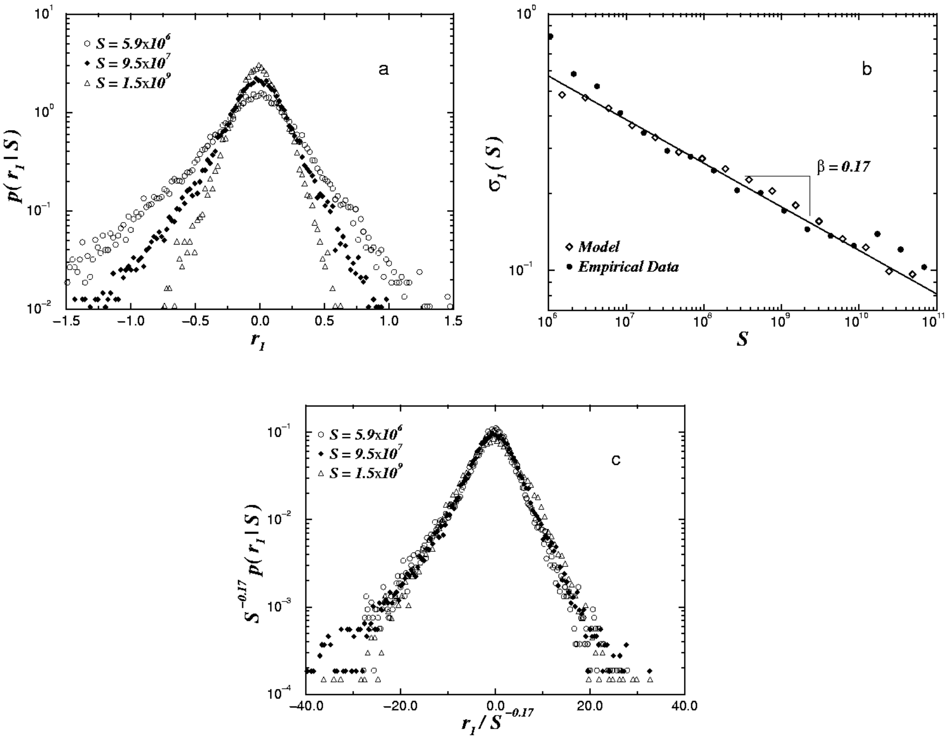

Figure 6. (a) Dependence of the standard deviation of the growth rates on firm size. Shown are the predictions of the model and the empirical results. The values of the parameters are the same as in Fig. 5. The straight line with slope 0.17 is a least square fit to the predictions of the model. (b) Probability density of one-year growth rates for different firm sizes plotted in scaled variables. The distributions are tent-shaped, as for the empirical data [6], and consistent with an exponential distribution. (c) Probability density of one-year growth rates for different firm sizes plotted in scaled variables.

subunits have similar sizes, we can apply the central limit theorem and Eq. (8), from which it follows that $\sigma_1 \sim \bar{N}^{-1/2}$, leading to the testable scaling law

$$\beta = (1 - \alpha)/2 \,. \tag{9}$$

For $\alpha = 0.66 \pm 0.05$, Eq. (9) predicts $\beta = 0.17 \pm 0.03$, in good agreement with our independent calculation of β.

6.2. DISCUSSION

We find that the predictions of the model are only weakly sensitive to the parameter values, which perhaps is the reason why firms operating in quite different industries are described by very similar empirical laws. Accordingly, we conjecture that the scaling laws found for US manufacturing firms [6] also hold for other countries, such as Japan, with $\beta \approx 0.2$; this conjecture has recently been confirmed [16].

The present model rests on a small number of assumptions. The three key assumptions are: (i) Firms tend to organize themselves into multiple divisions once they achieve a certain size. This assumption holds for many modern corporations [27]. (ii) There is a broad distribution of minimum scales in the economy. This assumption has also been verified empirically [26]. (iii) Growth rates of different divisions are independent of one another. For an economist, the latter is the stronger of the these assumptions. However, we find that correlations in the growth rates of divisions within a same firm, even weak correlations, lead to $\beta \to 0$. Thus, we confirm that it is the assumption of independence among the growth rates that generates results similar to the empirical findings of Refs. [6].

There are two features of our results that are perhaps surprising. First, although firms in our model consist of independent divisions, we do not find $\beta = 1/2$. To understand why $\beta < 1/2$, suppose that the distribution of $s_m \equiv \ln S_{\min}$ is a Dirac δ-function. Although this assumption is unrealistic, it leads to an understanding of the underlying mechanisms in the model. In this case, it is a plausible assumption that the number of divisions will increase linearly with firm size, because the distribution of division sizes is narrow and confined between $S_{\min}$ and $S_{\min}/V$. This hypothesis is confirmed numerically, and we find (i) $\beta = 1/2$ and $\alpha = 0$ and, (ii) that the distribution of the logarithm of firm sizes is still close to Gaussian, with a width $\mathcal{W}$ which is a function of the parameters of the model. Then, by integration of the distribution of the logarithm of firm sizes over s_m, we can estimate the value of β for the case of a broader distribution of s_m. Suppose that s_m follows some arbitrary distribution with width $\mathcal{D}$. Averaging $\sigma_1^2(S)$ over this distribution, we find $\beta = \mathcal{W}/2(\mathcal{D}+\mathcal{W})$. For a wide range of the values of the model's parameters, $\mathcal{D} > \mathcal{W}$, and we find that β is remarkably close to the empirical value $\beta \approx 0.2$.

Second, the distribution $p(r_1|S)$ is not Gaussian but "tent" shaped. We find this result arises from the integration of nearly-Gaussian distributions of the growth rates over the distribution of $S_{\min}$. For large values of $|r_1|$, the saddle point approximation gives $p(r_1|S) \sim \exp(-\log^2|r_1|)$, which decays slower than exponentially, in qualitative agreement with the model's predictions and with empirical observations. For $|r_1| \ll 1$, $p(r_1|S)$ is approximately Gaussian, while for intermediate values of $|r_1|$, the distribution decays exponentially. Our analytical predictions are in agreement with the model and with empirical results.

7. Concluding remarks: The growth of complex organizations

We study the growth dynamics of publicly-traded US manufacturing firms from 1974 to 1993. We find that the distribution of the logarithms of the growth rate decays exponentially. Furthermore, we observe that the standard deviation of the distribution of growth rates scales as a power law with the size S of the firm.

We consider a microscopic model for the growth dynamics of firms. The model leads to a number of conclusions. First, it suggests the deviations in the empirical data from predictions of the random multiplicative process may be explained (i) by the diversification of firms, i.e., firms are made up of interacting subunits; and (ii) by the fact that different industries have different underlying scales, i.e., there is a broad distribution of minimum scales for the survival of a unit (for example, a car manufacturer must be much larger than a software firm).

Second, the model suggests a possible explanation for the common occurrence of power law distributions in complex systems. Our results suggest that the empirically observed power law scaling does not require the system to be in the critical state, but rather can arise from an interplay between random multiplicative growth and the complex structure of the units composing the system. Here we addressed the case in which the interactions between the units can be treated in a "mean field" way through the imposition of a minimum size for the subunits. More general interactions may still lead to power law scaling, so our model may offer a framework for the the study of complex systems.

In fact, if the model proposed here is a reasonable description of the dynamic mechanisms for firm growth, then, due to their robustness, we expect the same empirical laws to be verified by the growth dynamics of other complex organizations. Such a possibility has been recently highlighted by the finding that the growth rates of the gross domestic product (GDP) of countries obey the same scaling laws as found for firms [62]. Thus, the results reported here might be relevant to the growth dynamics of other complex organizations.

Furthermore, our results support the possibility that the formalism and concepts used to describe complex but inanimate systems comprised of many interacting particles (as occurs in many physical systems) may be usefully extended to describe complex but animate systems comprised of many interacting subsystems (as occurs in economics).

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