

# HOW THE DYNAMICS OF THE FREE MARKET CREATES INFLATION AND UNEMPLOYMENT: A STABILITY ANALYSIS

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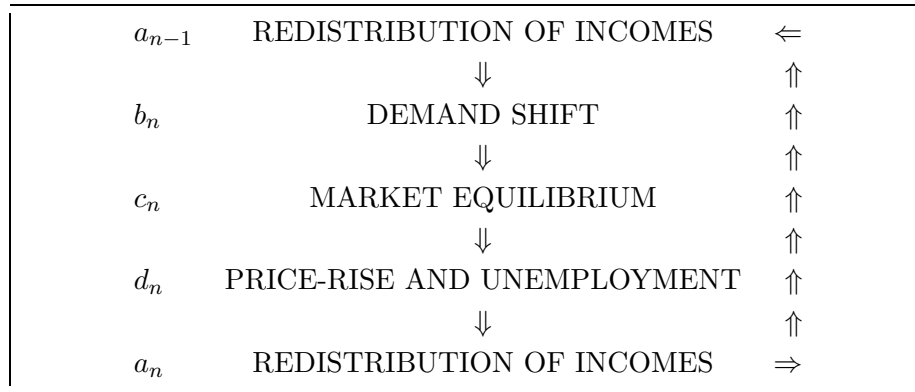
AND

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## 1. Introduction

In the preceding paper ?? one of us (EA) described a new macro-economic theory, “The Income Redistribution Theory of Inflation and Unemployment” (IRTIU), which is a dynamic theory describing an intrinsic economic instability. In this theory inflation and unemployment mutually feed each other and swell together through the inequitable redistribution of incomes, which they both generate. The dynamical process is described schematically in Fig. 1



*Figure 1.* The dynamic cycle of the income redistribution theory of inflation and unemployment

The economy is initially at or close to equilibrium. A disturbance redistributes incomes (at time  $a_{n-1}$ ). This causes a shift in the demand structure of the economy at time  $b_n$  raising demand in some markets and causing demand deficiency in others. Prices and transactions adjust to the new demand pattern ( $c_n$ ) causing both a price rise and unemployment ( $d_n$ ). This causes a new redistribution of incomes ( $a_n$ ). Evidently the original equilibrium can be stable or unstable depending on the relative sign of the new redistribution compared to the original disturbance.

Thus inflation and unemployment are regarded as an instability of the economic system, as manifestations of a spontaneous process, which is natural to the market and reflects the fact that the original market equilibrium was unstable. In this respect the IRTIU is in fairly sharp contrast to most theories of inflation and unemployment, which look for the reasons for such unpleasant economic developments outside of economics — in politics, in external shocks, in market imperfections, etc. — all of which inhibit the free market from reaching its proper equilibrium. (Examples could be supply shocks for whatever reason, but also misjudged government intervention, changes in the money supply, changes in the wage level . . .).

Our purpose here is to describe a simple formal model which shows the basic macroeconomic instability described by the IRTIU. We apply techniques which are fairly common in Physics to the modeling of a macroeconomic instability<sup>1</sup>. We will keep the model as simple as possible. The formal model we describe is, we believe, the simplest, which can still show the basic stagflationary instability in which we are interested. This means of course that many factors and features which appear and are important in real economies are omitted. This drastic simplification allows us to construct a model which can be fully analyzed with relatively simple tools. This makes the internal logic of the analysis clearer, more translucent and less obscure. Because the structure of the model is so simple, it is also clear that the cause of inflation and unemployment is indeed the macroeconomic market instability. The extraneous causes, errors and market imperfections to which they are frequently attributed by economists and economic policy makers simply do not appear in the model.

We note that it is fairly straightforward to make the formal model much more complete by adding realistic features and parameters and will hint at how this can be done. This is presumably necessary if one wants to do

<sup>1</sup>To a theoretical physicist it seems natural to expect that a system as complex as a national economy will have both stable and unstable equilibrium states connected by instabilities and also that one should use a theoretical apparatus which is flexible enough to investigate the consequences. This is what we try to do here. It should however be noted that many, if not most economist do not believe that this is feasible approach in their field.

econometric modeling. Evidently the analysis very rapidly becomes much more complex and can no longer be fully analyzed as a formal model.

## 2. The formal model

### 2.1. THE PARTIAL AGGREGATE INCOMES

We consider an economy for which the total aggregate income (i. e. ) the sum of all individual incomes) in time period  $n$  is  $Y^n$ . To analyze the effects of income distribution, we have to divide aggregate income. We divide it into the partial aggregate incomes of two income groups in the economy,

$$Y^n = Y_w^n + Y_g^n \quad (1)$$

The partial aggregate income of the group of wage earners,  $w$ , is determined by the wage level,  $W^n$  and by the level of employment,  $E^n$ :

$$Y_w^n = W^n E^n. \quad (2)$$

The partial income of the non-wage income group,  $g$ , can then be determined as a residue

$$Y_g^n = Y^n - Y_w^n. \quad (3)$$

### 2.2. THE CONSERVATION OF MONEY

In any real market the quantity of money is conserved in transactions. - that is as a flowing quantity. When **A** buys something from **B**, the amount which **A** pays is exactly equal to the amount which **B** receives. This is a basic conservation law of monetary economies. It distinguishes them from primitive barter economies. Nothing is conserved when I exchange tomatoes for apples, but when I buy apples money is conserved even after the apples are consumed. This conservation law has important implications in economics. In particular:

- **Say's law** which says that in *macro-economics*, for the economy as a whole, supply and demand are always exactly equal to each other. This law has been an important part of economics since Ricardo - its most recent application has probably been to the theoretical basis underlying *supply-side economics*.
- A closed economy cannot save up. Since for a closed economy money-incomes are equal to money-expenditures, reducing money-expenditures automatically implies an equal reduction in money-incomes. Keynes called this the **paradox of thrift**.

One would guess that this conservation law is probably also important in understanding the anomalous power laws observed so often - and discussed in many places in this volume.

For completeness we emphasize that the conservation of money is not an absolute conservation law. It is only a conservation law for money in its role in commercial transactions - as a flow quantity. Money is created and destroyed by governments, by central banks and in effectively created and destroyed by a variety of other financial agents and markets.

We want to consider a monetary economy, which does exhibit the basic conservation law. We achieve this by neglecting saving. We assume:

- that **all** income earned in period  $n$  is spent on consumption in the subsequent period  $n + 1$ , and also that
- No new money is created.

It then follows that the aggregate income cannot change:

$$Y_w^{n-1} + Y_g^{n-1} = Y_w^n + Y_g^n; \quad Y^{n-1} = Y^n = Y. \quad (4)$$

For convenience, we will also assume that nominal wages do not change, so that

$$W^{n-1} = W^n = W. \quad (5)$$

### 2.3. DIFFERENTIATED AGGREGATE DEMANDS

The distribution of incomes between  $w$  and  $g$  can affect the economy only if the two income groups have different demand patterns, so that a redistribution of incomes can cause a change in the structure of demand. To describe this, we disaggregate the markets into two markets, the basic-goods market B, and the luxury-goods market L.

- The wage earners,  $w$  spend **all** their income in the B market and are the only buyers in this market, so that

$$Y_w^{n-1} = p^n Q_B^n \quad (6)$$

and

- $g$  spend all their income in the L market, so that

$$Y_g^{n-1} = p^n Q_L^n, \quad (7)$$

where  $p^n$  is the price for the time period  $n$  and  $Q_B^n$  and  $Q_L^n$  are the quantities produced and sold in the B and L markets, respectively.

This means that the differentiated demands in the two markets in the period  $n$  are hyperbolic

$$D_B^n = \frac{Y_w^{n-1}}{p}; \quad D_L^n = \frac{Y_g^{n-1}}{p}. \quad (8)$$

#### 2.4. DIFFERENTIATED SUPPLIES

To establish market equilibrium, we have to compare the differentiated demands Eq. (8) to supply, which is determined by the cost curves for the same two markets. Equilibrium will determine the price level,  $p^n$ , and the two quantities  $Q_B^n$  and  $Q_L^n$ .

### 3. Market equilibrium

#### 3.1. PRICE AND QUANTITY ADJUSTMENT

Quite generally markets will not react symmetrically to increasing and to decreasing demand. Prices increase more when demand rises than they decrease when demand falls. It is relatively easy to raise prices in response to increasing demand and relatively difficult and slow to increase production to accommodate the same demand. On the other hand, lowering prices in the face of already decreasing demand is difficult and often not even feasible — in particular, when the change in demand reflects a demand shift, so that there is also a parallel and profitable excess demand market in the same economy. We thus expect that markets will predominantly react to excess demand by price adjustment (raising prices) and to deficient demand by reducing production - quantity adjustment.

#### 3.2. MARKET CAPACITY

We incorporate this into our model by assuming inverted-L shaped supply curves,  $S_L$ ,  $S_B$ . The L market has a maximum capacity  $Q_{L0}$  and the B market a capacity  $Q_{B0}$ , which reflect the state of existing technologies and, largely the past history of the specific economy. It follows that

$$Q_L^n \leq Q_{L0}; \quad Q_B^n \leq Q_{B0}. \quad (9)$$

We assume that in the “short run”, that is on the time scales relevant for the dynamics, these capacities cannot increase.

We want to study developments when this equilibrium is disturbed by an initial ‘shock’, which shifts income from one group to the other.

We first consider the situation when the initial income-shock was inequitable, shifting income from the wage earners,  $w$ , to the profit earners,  $g$ . This creates excess demand in the L market and deficient demand in the B market<sup>2</sup>.

<sup>2</sup>This assumes implicitly that both markets were initially adjusted to an equilibrium states of the economy and were operating close to their capacity in this state.

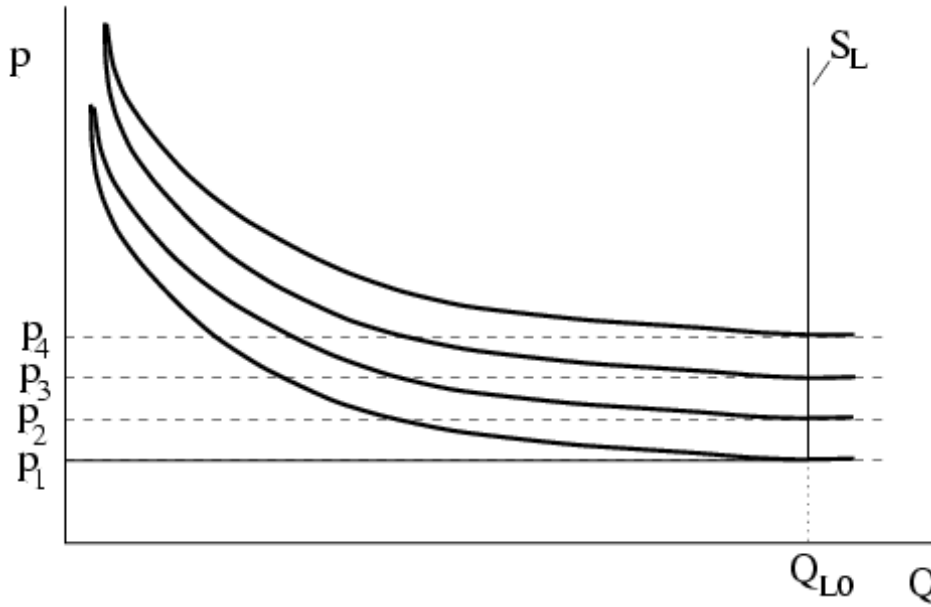


Figure 2. Supply and Demand in the L Market The L shaped supply function  $S_L$  and the demand function  $D_L, n(p) = Y_g^n/p$  for the L market. Under these conditions there is excess demand and the market reacts by pure price adjustment.

The response of the excess demand market L, is described in Fig. 2. The quantity is constant

$$Q_L^n = Q_{L0}. \tag{10}$$

The price  $p^n$  is then determined by the differentiated demand, Eq. 8,

$$p^n = \frac{Y_g^{n-1}}{Q_{L0}}. \tag{11}$$

In the B market there is deficient demand

$$Q_B^n = \frac{Y_w^{n-1}}{p^n}. \tag{12}$$

and  $Y_w^{n-1}$  is not sufficient to buy the full quantity  $Q_{B0}$ .

The establishment of market equilibrium in period  $n$  in response to the incomes

$$Y_w^{n-1}; \quad Y_g^{n-1} \tag{13}$$

in the preceding period, Eqs. 12 and 12 amounts to the determination of the three quantities

$$Q_L^n (= Q_{L0}), \quad Q_B^n \text{ and } p^n.$$

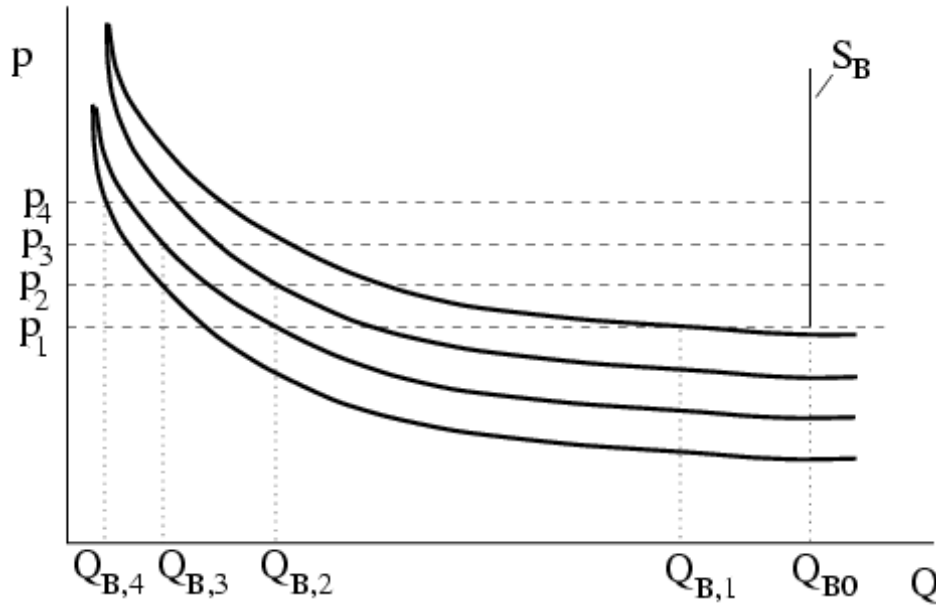


Figure 3. Supply and Demand in the B market The horizontal supply function *at the level* describes pure quantity adjustment.

### 3.3. EMPLOYMENT AND WAGE INCOME

To calculate incomes in the period  $n$  we first determine employment

$$E^n = \mu_B Q_B^n + \mu_L Q_L^n \quad (14)$$

where  $\mu_B$  and  $\mu_L$  are technological factors which describe productivity in the two markets.

It follows that the income from wages in period  $n$  is

$$Y_w^n = W E^n = W(\mu_B Q_B^n + \mu_L Q_L^n). \quad (15)$$

This allows us to close the dynamic cycle we described schematically in Fig. 1.

## 4. Dynamic mapping, fixed points and instabilities

We have four basic equations:

The conservation law, Eq. (4):

$$Y_w^{n-1} + Y_g^{n-1} = Y_w^n + Y_g^n = Y, \quad (16)$$

the market equilibrium equations, Eq. (11):

$$p^n = \frac{Y_g^{n-1}}{Q_{L0}} \quad (17)$$

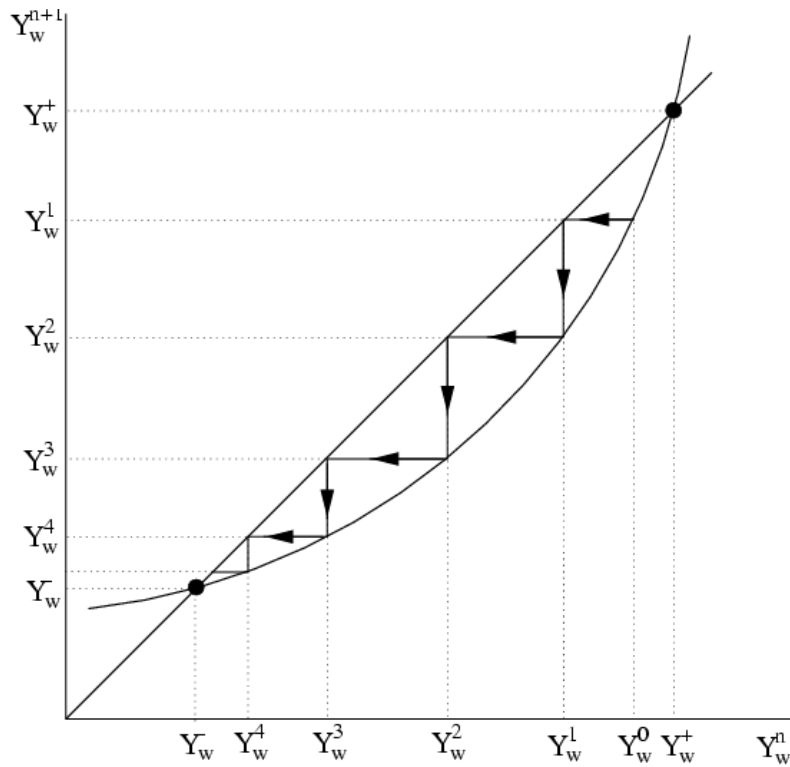


Figure 4. Plot of the wage-share,  $Y_w^{n+1}$  as a function of  $Y_w^n$  according to Eq. (23). The system starts at  $Y_w^0$  somewhat smaller than the “equitable” equilibrium value  $Y_w^+$ , and evolves gradually, through steps  $Y_w^1, Y_w^2, \dots, Y_w^n, \dots$  towards the stable equilibrium  $Y_w^-$ .

and Eq. (12):

$$Q_B^n = \frac{Y_w^{n-1}}{p^n} \tag{18}$$

and the equation which determines wage income, Eq. (15):

$$Y_w^n = W(\mu_B Q_B^n + \mu_L Q_L^n). \tag{19}$$

The four equations (4–19) define a dynamic mapping, which maps the state of the economy in period  $n - 1$  on its state in period  $n$ . With all the simplifications we have introduced, this can be regarded as a single parameter mapping — for example as a dynamical equation for the income from wages

$$Y_w^{n+1} = Y_w^{n+1}(Y_w^n) = W Q_{L0} \left( \mu_L + \mu_B \frac{Y_w^n}{Y - Y_w^n} \right) \tag{20}$$



This is described graphically in Fig. 4. It depends on the six model parameters, which we have set constant

$$Y, W, Q_{L0}, Q_{B0}, \mu_B, \mu_L \quad (21)$$

The mapping also determines the evolution of the remaining dynamic variables of the model:

$$p^n, Q_L^n, Q_B^n, Y_g^n, E^n \quad (22)$$

The mapping (20) has two fixed points obtained by equating  $Y_w^{n+1}$  and  $Y_w^n$  in Eq. (20)

$$Y_w^\pm = Y_w^{n+1}(Y_w^\pm) = WQ_{L0} \left( \mu_L + \mu_B \frac{Y_w^\pm}{Y - Y_w^\pm} \right). \quad (23)$$

These fixed points are the equilibrium states. Thus our model has two equilibrium states rather than one. Of these two equilibria the upper equilibrium '+' represents a more equitable income distribution

$$Y_w^+ \geq Y_w^-. \quad (24)$$

But this equilibrium state is unstable. It decays to the less equitable equilibrium state '-'. In the process, the wage share in  $Y$  decreases continuously

$$Y_w^+ \geq Y_w^0 \geq Y_w^1 \geq Y_w^2 \geq \dots \geq Y_w^n \geq \dots \geq Y_w^-. \quad (25)$$

At the same time employment also decreases, because

$$E^n = \frac{Y_w^n}{W} \quad (26)$$

and unemployment grows.

Instead of deriving prices from this, we can also consider the mapping of the price level directly

$$p^{n+1} = p^{n+1}(p^n) = \frac{Y}{Q_{L0}} + W(\mu_B - \mu_L) - \frac{W\mu_B Y}{p^n Q_{L0}} \quad (27)$$

which is described in Fig. 5.

This describes an inflationary process — again associated with the decay from the unstable '+' fixed point to the high price stable equilibrium '-'.

## 5. Conclusions

We have developed a very simple macroeconomic model and analyzed its dynamical stability.

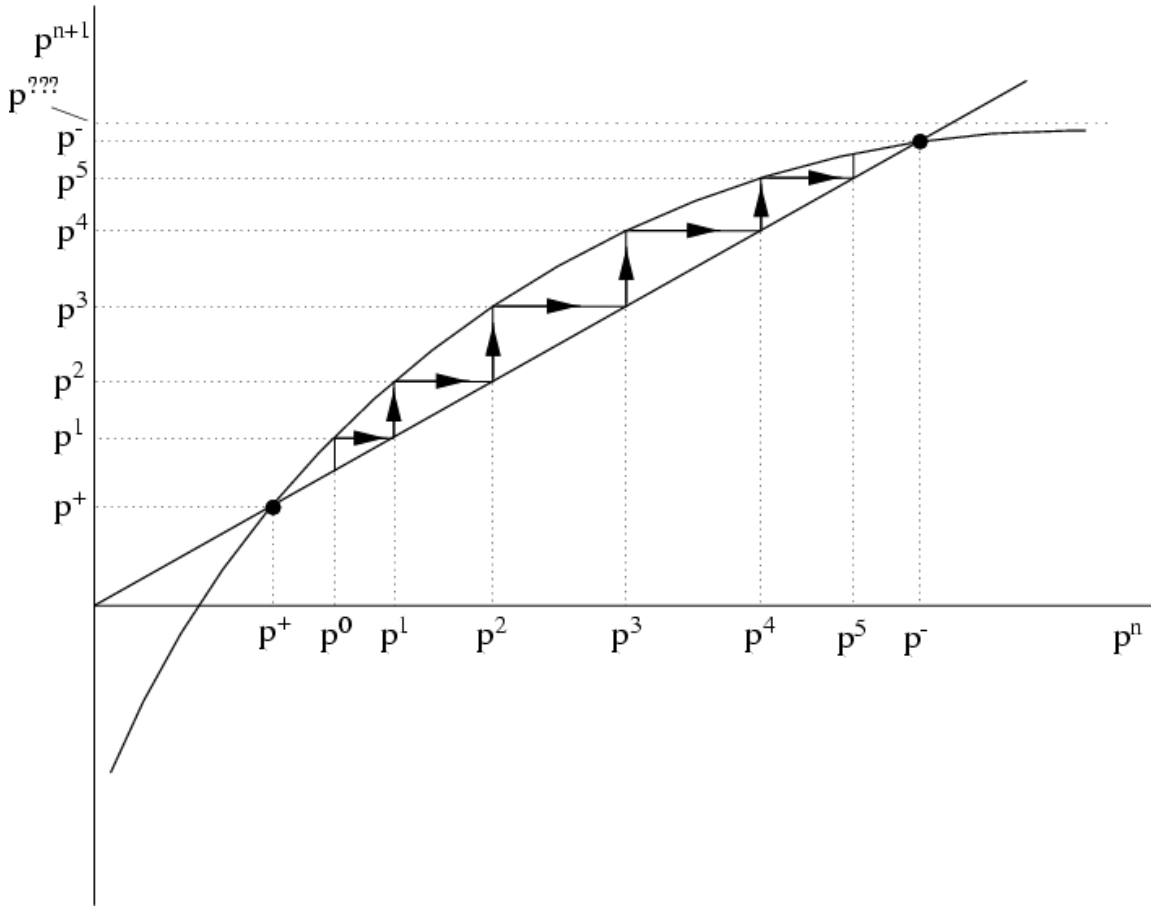


Figure 5. The dynamic instability described as inflation — the time evolution of prices from the unstable equilibrium state '+' to the stable equilibrium '-'.

We have modified the standard macroeconomic formalism just sufficiently to allow us to incorporate the effects of the spontaneous redistribution of incomes by macro-economic processes.

Evidently, one cannot study these effects without first disaggregating the macroeconomic variables. This required the introduction of Partial Aggregates to describe income distribution, and Differentiated Aggregates to describe the response of markets to demand shifts.

We have also incorporated the basic conservation law of a monetary economy, the conservation of money in transactions into our model.

We have found that our model economy had two equilibrium states rather than one. It decays spontaneously from a relatively equitable income distribution equilibrium with low prices and low unemployment to a

stable, inequitable equilibrium with high unemployment and a high prices, this basic instability is a stagflationary instability in which inflation and unemployment develop as parts of the same macroeconomic instability.

We have discussed some of the implicit and explicit assumptions as they appeared. We think they are plausible, but, obviously, we could not discuss their justification fully.

It seems important to emphasize one additional point. Our model is a purely “short term” model and, thus follows Keynes’ “General Theory”. We do not allow the markets to readjust to changing demand by changing the defining parameters of the market ( $Q_{L0}$ ,  $Q_{B0}$ ). This is of course convenient and simplifies the analysis. However, in the context of a dynamical model it is really only an assumption about relative rates. It assumes that the dynamic instability proceeds rapidly. The long term effects cannot then develop and prevent this because conditions change continually. Investment can presumably catch up when the stable equilibrium state is reached, so that the structure of the markets can adjust to this new distribution of incomes. It is interesting to note that this is again a destabilizing effect. Once the markets have adjusted, the income distribution again becomes unstable.

## Appendix

In the discussions above we have **assumed** that there was excess demand in the L market and deficient demand in the B market. This was attributed to the initial disturbance which destabilized the initial equilibrium. Evidently, this is not always the case. One can, of course, also consider the reverse situation - excess demand in the B market arising, for example, from a ‘wage shock’ which raised wages too high.

It is straightforward to see that the mapping (27) is replaced by

$$p^{n+1} = p^{n+1}(p^n) = W(\mu_B - \mu_L) + W\mu_B \frac{Y}{p^n Q_{B0}} \quad (28)$$

which only has one unique fixed point for which the price is positive

$$p^{\text{equ}} = \frac{W(\mu_B - \mu_L)}{2} \left[ \mp 1 + \sqrt{1 + \frac{4\mu_B Y}{(\mu_B - \mu_L)^2 W Q_{B0}}} \right]. \quad (29)$$

This fixed point is always stable, and the dynamics is described in Fig. 6.

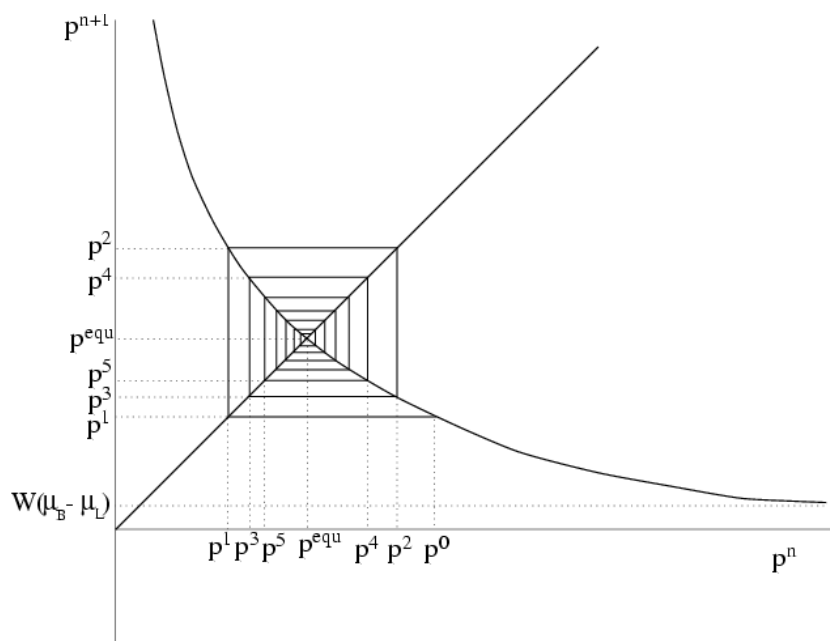


Figure 6. The time evolution of the prices when there is excess demand in the B market and deficient demand in the L market