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BREAK-DOWN OF SCALING AND CONVERGENCE TO GAUSSIAN DISTRIBUTION IN STOCK MARKET DATA

L. KULLMANN and J. KERTÉSZ*
Department of Theoretical Physics, Technical University of Budapest, Budafoki ut 8, H-1111, Budapest, Hungary
J. TÖYLI and K. KASKI
**Laboratory of Computational Engineering, Helsinki University of Technology, P. O. Box 9400, FIN-02015 HUT, Finland*
A. KANTO
Department of Economics, Helsinki School of Economics and Business Administration, FIN-00100 Helsinki, Finland

We analyze the Standard and Poor's 500 index data of the New York Stock Exchange for more than 32 years. It was suggested earlier that the high frequency data are well described by a truncated Lévy distribution and scaling with respect to the sampling time differences was found. The truncated character of the distribution implies that scaling must break down and that the distribution ultimately converges to a Gaussian. We show by comparing Lévy and Gaussian fits that the characteristic time of the break-down of scaling is of the order of few days. The analysis of the dependence of the kurtosis on the time differences shows that this is much shorter than the time needed for the *convergence* to the Gaussian being of the order of months.

1. Introduction

Mantegna and Stanley analyzed high frequency data of the absolute returns of the S&P500 index and found that a *truncated Lévy* distribution fits well the empirical data and the curves obtained by different time intervals can be scaled together[1]. Since the truncated Lévy is not a stable distribution, the observed scaling must be limited, asymptotically the distribution has to cross over to a Gaussian, provided the independence of the data can be assumed. We argue that this crossover is slow, and it can be characterized by at least two times, one of them showing the time in which the scaling breaks down and the other, which is several orders of magnitude larger, indicating the convergence to the Gaussian distribution.

2. Time scale of the break-down of scaling

Earlier we analyzed[2] the Standard & Poor's stock index data between 1962 and 1995. To study the scaling behavior we fitted the Lévy distribution to the distribution of the logarithmic return. By definition of the stability the scaling holds only if the Lévy exponent is a constant function of the time difference. We see (Fig. 1.a.) that it is not true if the time difference is larger than one day, while the result by

Mantegna and Stanley[1] showed that scaling holds for shorter time difference than one day. Therefore the characteristic time of the break-down of scaling τ_s is in the range of one day.

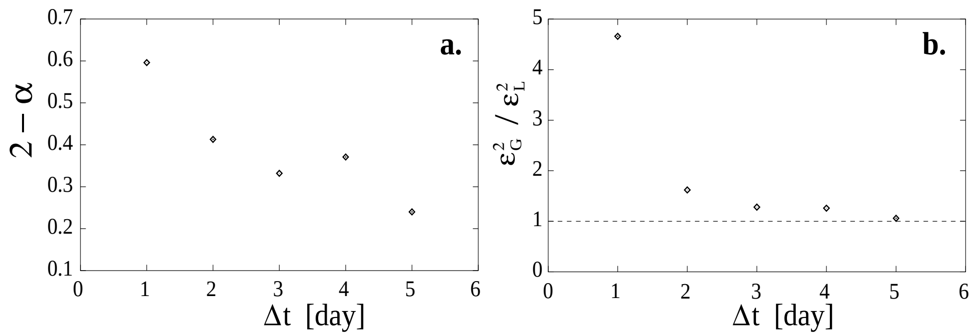

Fig. 1. Demonstration of the break down of the scaling. **a)** the exponent **b)** the error quotients as the function of the time difference.

3. Convergence to Gaussian distribution

The fact that the scaling breaks down if the time difference is larger than one day does not mean that the distribution becomes Gaussian on this scale. Fig.1.b. shows that the error quotients of the fits tend to unity in this time region, which only means that the Lévy distribution does not fit better than the Gaussian. One can define several characteristic times, one as the error of the fit τ_s; for which we get the value approximately one day, and the other one τ_G which sets the scale of the convergence to the Gaussian distribution. This latter can be defined for example by using the convergence of the kurtosis[3]. If we assume that the distribution of the logarithmic return is a truncated Lévy, then the kurtosis of the sum of such variables can be calculated analytically (3.1). It will be a function of the exponent α, the scale factor a_α, and the truncation parameter μ. We assumed here, for simpler calculation, that the cut-off of the distribution is exponential, however recent studies seem to yield a higher order power law[4]. We think that these do not change the result substantially (and possibly only increase the characteristic time τ_G).

$$\kappa(N) = \frac{M_4}{M_2^2} = 3 + \frac{1}{N}\left[-\frac{\cos\left(\frac{\pi\alpha}{2}\right)}{a_\alpha}\,\frac{(\alpha-2)(\alpha-3)}{\alpha(\alpha-1)}\,\mu^{-\alpha}\right] \tag{3.1}$$

Replacing the parameters in Eq. (3.1) –their values can be obtained from the truncated Lévy fit on the empirical data– the so obtained kurtosis as a function of the time difference ($N = \Delta t$) fits well the measured values calculated directly from the raw data, see Fig. 2.

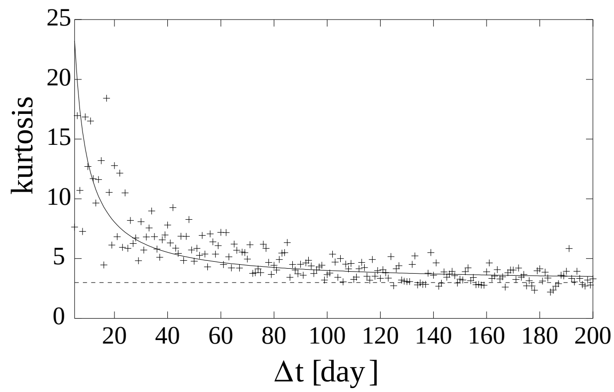

Fig. 2. The convergence of the kurtosis. The full line is the calculated value of the kurtosis with $\alpha = 1.4$, $a_\alpha = 8.6$, $\mu = 0.009$ taken from the fit.

4. Conclusion

We have studied the characteristic times in the logarithmic return of the distribution of the stock index. We have concluded that at least two different characteristic times can be defined: One of them (τ_s) shows the break-down of scaling, the other one (τ_G) can be defined as the time scale of the convergence of the kurtosis. We found that τ_s is of the order of one day while τ_G is in the range of few months, i.e. $\tau_s \ll \tau_G$. We would like to point out that this result is a consequence of the initial truncated Lévy distribution and no further assumptions for example on multiscaling were involved.

Acknowledgement

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