Class 6 - Waveguides

Class material

Exercise 6.1 - TE modes in a rectangular wavewuide

Give the electric and magnetic fields of the transverse electric (TE) modes propagating in a wavegoude with rectangular cross-section with a and b sidelengths! The walls of the waveguide considered as an ideal conductor. Give the group and phase velocity!

(a) Write the equation of motion for the electric and magnetic field and the boundary conditions!

(b) Solve the equations for the case of TE modes!

(c) Determine the group and phase velocity!

Exercise 6.2 - TM modes in a circular waveguide

Give the electric and magnetic fields for the transverse magnetic (TM) modes propagating in a waveguide with circular cross-section! Determine the Poynting vector!

(a) Write the equation of motion for the electric and magnetic field and the boundary conditions!

(b) Solve the equations for the case of TM modes in polar coordinates!

(c) Give the Poynting vector!

Exercise 6.3 - TEM modes in a coaxial wire

Investigate the possibility of the formation of the TEM modes in a coaxial wire with outer radii R and inner radii r!

(a) Determine the differential equation for the TEM modes!

(b) Solve the equations!

Homework

The following problems (marked with an asterisk) form the basis of the short test at the beginning of the next class.

Exercise 6.4 - Square waveguide*

Given an ideal waveguide($\sigma = \infty$) with square cross-section.

(a) Give the Poynting vector for the TE and TM modes!

(b) Give the parallel and perpendicular component of the Poynting vector to the axis of the waveguide!

Exercise 6.5 - Cut-off frequencies in a circular waveguide*

Given an ideal waveguide with circular cross-section. Give the two lowest cut-off frequency mode!

(a) For the TE modes.

(b) For the TM modes

(c) Show that the TE_{01} and the TM_{11} modes have the same cut-off frequency!

Exercise 6.6 - Coaxial waveguide*

Given a circular coaxial wire with core radii a and shield radii $b = 2a$.

(a) Give the cut-off frequency of the TEM mode!

(b) Give the lowest cut-off frequencies for the TE and TM modes!

These problems are for further practice and to have some fun!

Exercise 6.7 - A waveguide with quarter round cross-section

Given an ideal waveguide with quarter-round cross-section. Give the cut-off frequencies for the lowest order TE and TM modes!

Exercise 6.8 - (Jackson 8.2)

A transmission line consisting of two concentric circular cylinders of metal with conductivity σ and skin depth δ, as shown, is filled with a uniform lossless dielectric (ϵ, μ). A TEM mode is propagated aling this line.

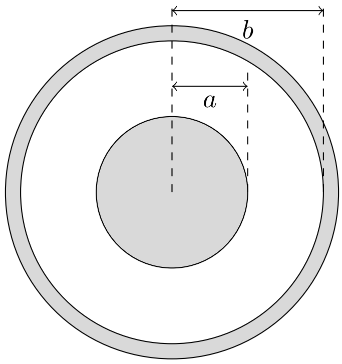

Figure 1

(a) Show that the time-averaged power flow along the line is

$$P = \sqrt{\frac{\mu}{\epsilon}} \pi a^2 |H_0|^2 \ln\left(\frac{b}{a}\right)$$

where H_0 is the peak value of the azimuthal field at the surface of the inner conductor.

(b) Show that the transmitted power is attenuated along the line as

$$P(z) = P_0 \,\mathrm{e}^{-2\gamma z}$$

where

$$\gamma = \frac{1}{2\sigma\delta}\sqrt{\frac{\epsilon}{\mu}} \frac{1/a + 1/b}{\ln(b/a)}$$

(c) The characteristic impedance Z_0 of the line is defined as the ratio of the voltage between the cylinders to the axial current flowing in one of them at any position z. Show that for this line

$$Z_0 = \frac{1}{2\pi}\sqrt{\frac{\mu}{\epsilon}} \ln(b/a)$$

(d) Show that the series resistance and inductance per unit length of the line are

$$R = \frac{1}{2\pi\sigma\delta}(1/a + 1/b)$$
$$L = \left\{\frac{\mu}{2\pi} \ln(b/a) + \frac{\mu_c \delta}{4\pi}(1/a + 1/b)\right\}$$

where μ_c is the permeability of the conductor. The correction to the inductance comes from the penetration of the flux into the conductors by a distance of order δ.

Exercise 6.9 - (Jackson 8.3)

(a) A transmission line consists of two identical thin strips of metal, shown in cross section in the sketch. Assuming that $b \gg a$, discuss the propagation of a TEM mode on this line, repeating the derivations of Exercise 6.8. Show that

$$P = \frac{ab}{2}\sqrt{\frac{\mu}{\epsilon}}|H_0|^2 \,, \qquad \gamma = \frac{1}{a\sigma\delta}\frac{\epsilon}{\mu} \,,$$
$$Z_0 = \sqrt{\frac{\mu}{\epsilon}}\left(\frac{a}{b}\right) \,, \qquad R = \frac{2}{ab\sigma} \,, \qquad L = \left(\frac{\mu a + \mu_c \delta}{b}\right)$$

where the symbols on the left has the same meaning as in Exercice 6.8.

(b) The right side of the figure shows the cross section of a microstrip line with a strip of width b mounted on a dielectric substrate of thickness h and dielectric constant ϵ, all on a ground plane. What differences occur here compared to part (a) if $b \gg h$? If $b \ll h$?

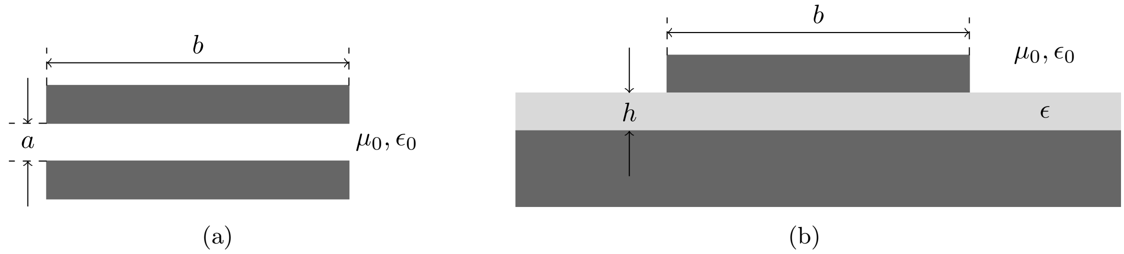

Figure 2

Exercise 6.10 - (Jackson 8.4)

Transverse electric and magnetic waves are propagated along a hollow, right circular cylinder with inner radius R and canductivity σ.

(a) Find the cutoff frequencies of the various TE and TM modes. Determine numerically the lowest cutoff frequency (the dominant mode) in terms of the tube radius and the ratio of cutoff frequencies of the next four higher modes to that of the dominant mode. For this part assume that the conductivity of the cylinder is infinite.

(b) Calculate the attenuation constants of the waveguide as a function of frequency for the lowest two distinct modes and plot them as a dunction of frequency.

Exercise 6.11 - (Jackson 8.5)

A waveguide is constructed so that the cross section of the guide forms a right triangle with legs a, as shown. The medium inside has $\mu_r = \epsilon_r = 1$.

(a) Assuming infinite conductivity for the walls, determine the possible modes of propagation and their cutoff frequencies.

(b) For the lowest modes of each type calculate the attenuation constant, assuming that the walls have large, but finite, conductivity. Compare the result with that for a square guide of side a made from the same material.