



M Ű E G Y E T E M 1 7 8 2

# **Theoretical study of a domain wall through a cobalt nano-contact**

**László Balogh, Krisztián Palotás, Bence Lazarovits,  
László Udvardi, László Szunyogh**

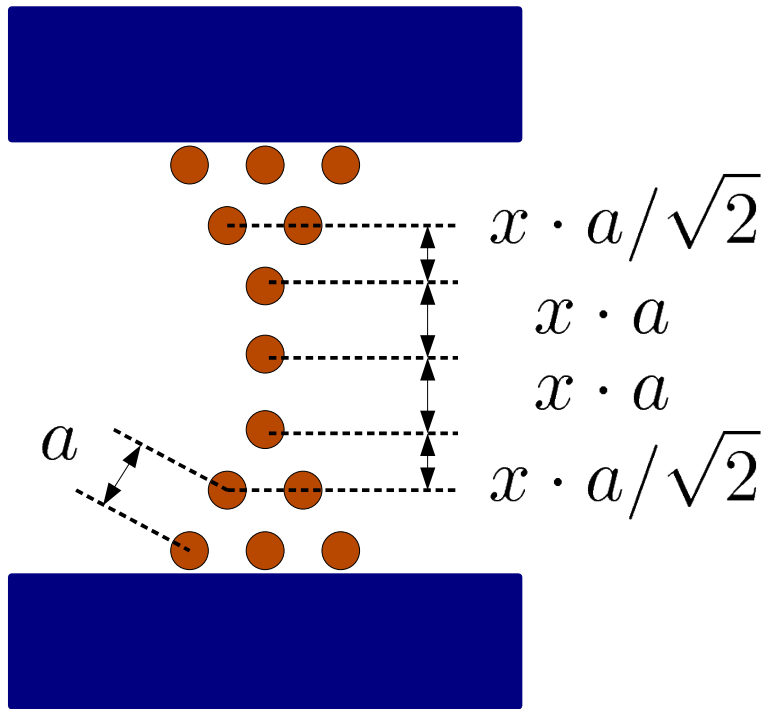
**Sovata, June 1–4, 2011.**

# Contents

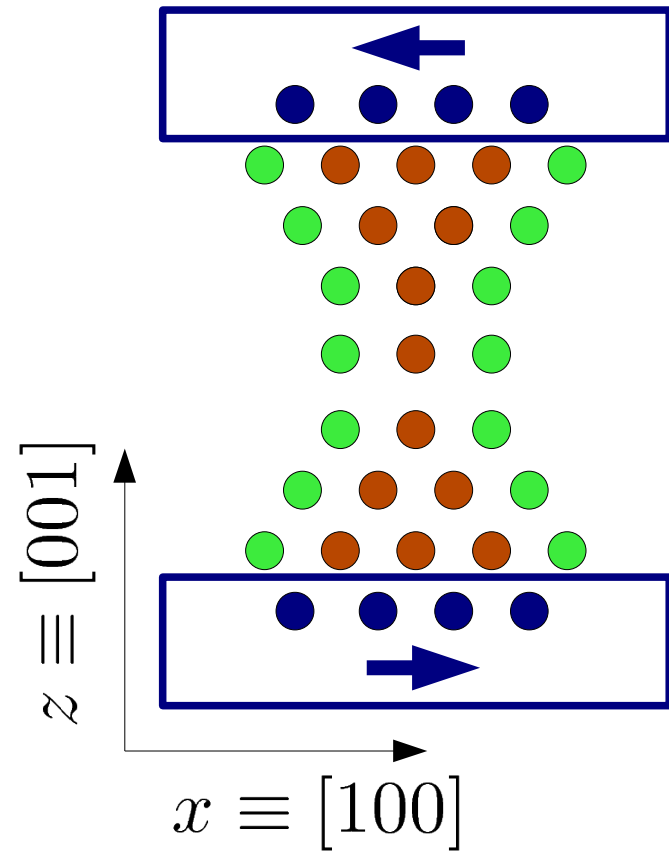
- Structure of the contact
- Modeling methods
- Relativistic effects
  - rotation around the [100] axis
  - decomposition of the band energy ( $E_b$ ) on the central atom

# Physical system:

Co[001] + 3 x 3 + 2 x 2 + 1 + 1 + 1 + 2 x 2 + 3 x 3 + Co[001]



$$x \in \{0.85, \dots, 1.15\}$$



# Isotropic Heisenberg model

$$\mathcal{H} = \frac{1}{2} \sum_{i \neq j} J_{ij} \boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j$$

Invariant under global spin rotation.

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Boundary condition:  $[100]$  and  $[\bar{1}00]$

Invariant under global spin rotation  
around the  $[100]$  axis.

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***Configurations rotated around the  $[100]$  axis are equivalent.***

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# Deficiencies of the isotropic Heisenberg model

Higher order terms can appear, e.g.,

$$J_{\square} [(\sigma_1 \sigma_2) (\sigma_3 \sigma_4) + (\sigma_2 \sigma_3) (\sigma_4 \sigma_1) + (\sigma_1 \sigma_3) (\sigma_2 \sigma_4)]$$

(see: S. Lounis, P. H. Dederichs, Phys. Rev. B. **82**, 180404 (2010))

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$$\begin{array}{l} \mathbf{J}_{ij} \begin{array}{l} \nearrow \\ \rightarrow \\ \searrow \end{array} \begin{array}{l} \mathbf{J}_{ij}^{\text{I}} = \left( \frac{1}{3} \text{Tr } \mathbf{J}_{ij} \right) \mathbf{I} \\ \mathbf{J}_{ij}^{\text{S}} = \frac{1}{2} (\mathbf{J}_{ij} + \mathbf{J}_{ij}^T) - \mathbf{J}_{ij}^{\text{I}} \\ \mathbf{J}_{ij}^{\text{A}} = \frac{1}{2} (\mathbf{J}_{ij} - \mathbf{J}_{ij}^T) \end{array} \end{array}$$

On-site anisotropy, e.g.,  $K(\theta) = -K_2 \sin^2(\theta)$

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***Demonstrations of the relativistic effects:***

♣ ***energy while rotating the whole configuration around the [100] axis.***

♣ ***expansion of the energy of the most symmetric atom in terms of the real spherical harmonics***

# Modeling methods I.

Screened Korringa–Kohn–Rostoker /  
embedded cluster method <sup>1</sup>

Infinitesimal rotations method: <sup>2</sup>

- ▶ parameters of isotropic Heisenberg model

Monte Carlo: simulated annealing /  
Metropolis algorithm

- ▶  $\approx$  ground state

Minimization of the band energy by Newton–  
Raphson method: an iterative minimum  
searching algorithm

- ▶ ground state magnetic configuration
- ▶ potential ( $V, B_x$ ) at this configuration

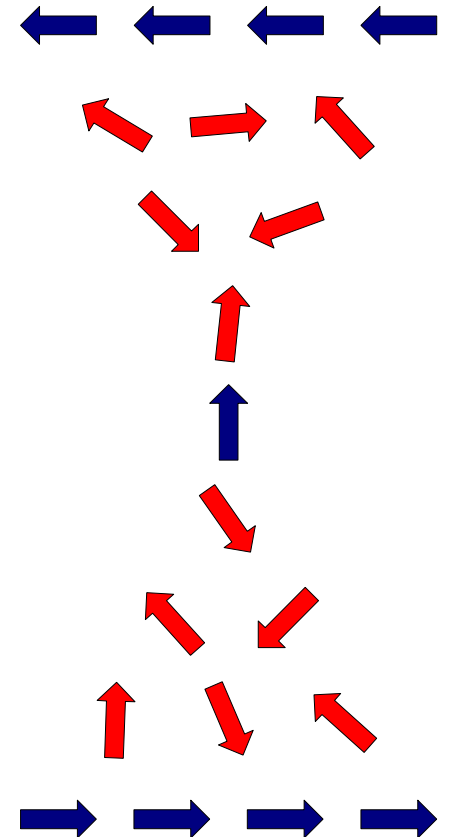
Embedding + band energy calculation  
+ magnetic force theorem

- ▶ energy functions

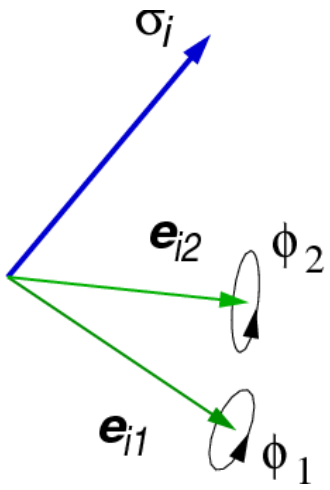
<sup>1</sup> B. Lazarovits, *et al.*  
PRB **65**, 104441 (2002)

<sup>2</sup> A.I. Liechtenstein *et al.*  
JMMM **67**, 65 (1987)

<sup>2</sup> L. Udvardi *et al.*  
PRB **68**, 104436 (2003)



# Modeling methods II.



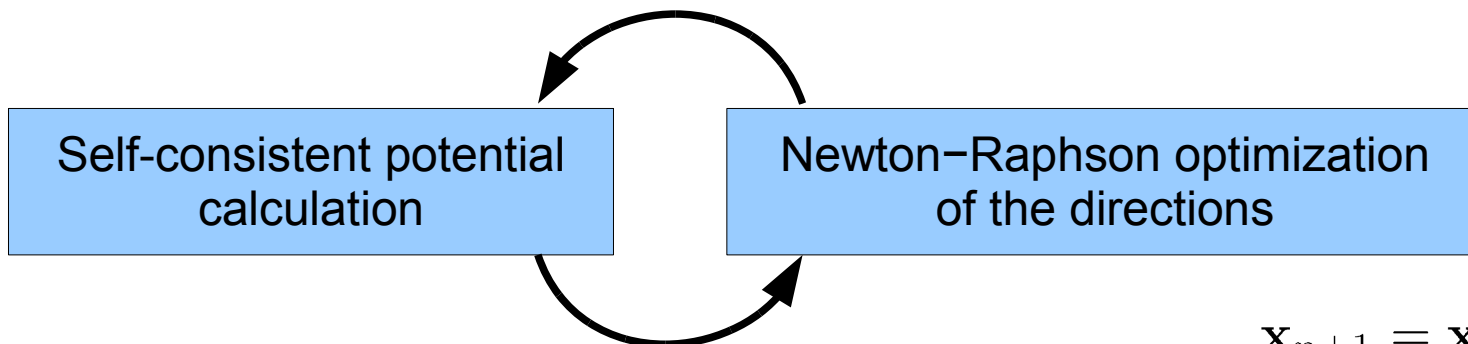
$$E_b = \int_{-\infty}^{E_F} (\varepsilon - E_F) n(\varepsilon) d\varepsilon = -\frac{1}{\pi} \text{Im} \int_{-\infty}^{E_F} \text{Tr} \ln \tau(\varepsilon) d\varepsilon$$

$$\frac{\partial E_b}{\partial \phi_{i\alpha}} = \frac{1}{\pi} \text{Im} \int_{-\infty}^{E_F} i \text{Tr} \left\{ \tau_{ii}(\varepsilon) [\mathbf{e}_{i\alpha} \cdot \mathbf{J}, t_i^{-1}(\varepsilon)] \right\} d\varepsilon$$

$$\frac{\partial^2 E_b}{\partial \phi_{i\alpha} \partial \phi_{j\beta}} = \delta_{ij} \frac{1}{\pi} \text{Im} \int_{-\infty}^{E_F} \text{Tr} \left\{ \tau_{ii}(\varepsilon) [\mathbf{e}_{i\alpha} \mathbf{J}, [\mathbf{e}_{i\beta} \mathbf{J}, t_i^{-1}(\varepsilon)]] \right\} d\varepsilon$$

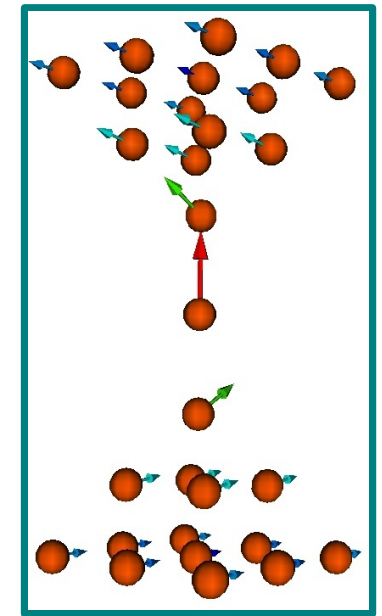
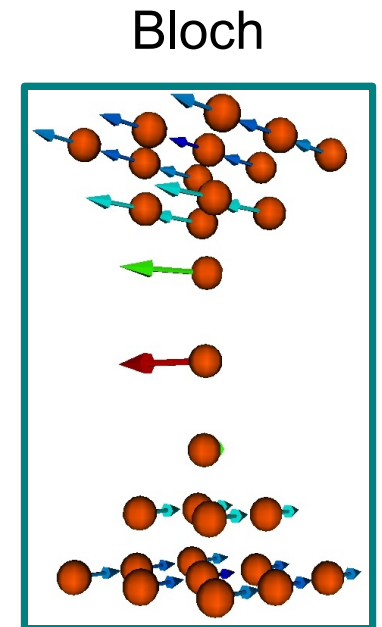
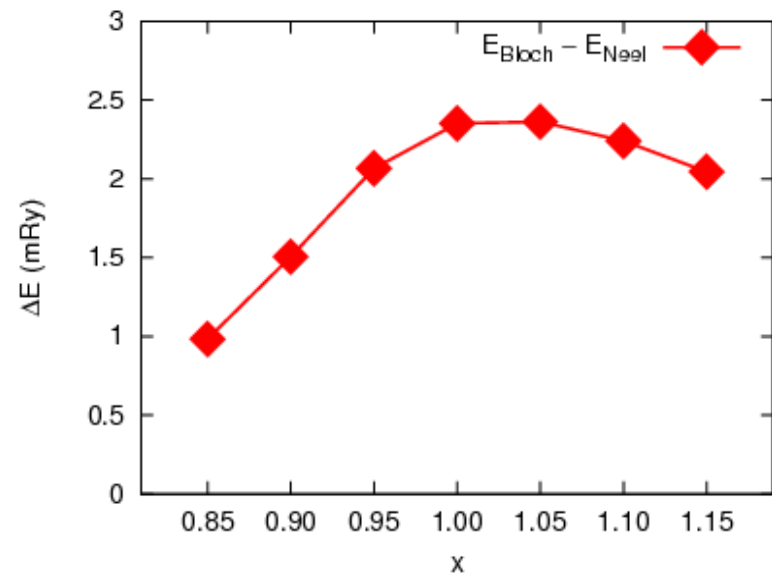
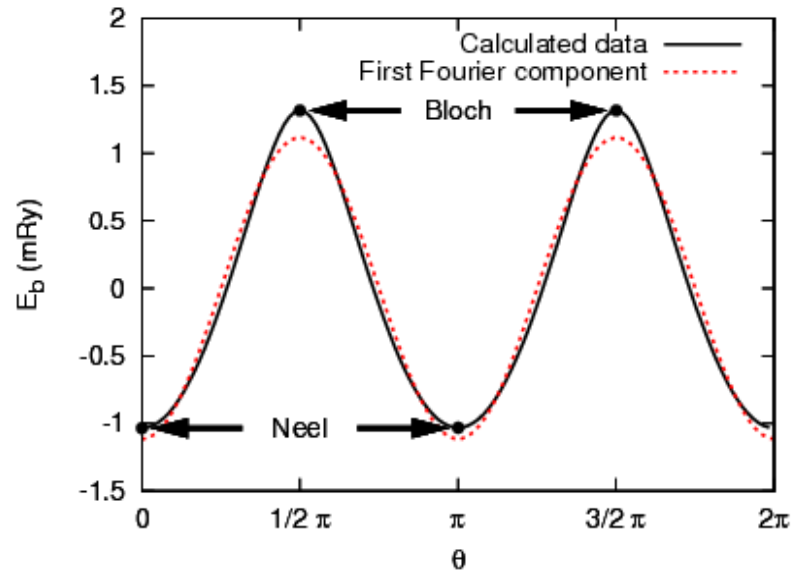
$$- \frac{1}{\pi} \text{Im} \int_{-\infty}^{E_F} \text{Tr} \left\{ \tau_{ij}(\varepsilon) [\mathbf{e}_{j\beta} \mathbf{J}, t_j^{-1}(\varepsilon)] \tau_{ji}(\varepsilon) [\mathbf{e}_{i\alpha} \mathbf{J}, t_i^{-1}(\varepsilon)] \right\} d\varepsilon$$

based on L. Udvardi *et al.* PRB **68**, 104436 (2003)



$$\mathbf{x}_{n+1} = \mathbf{x}_n - \frac{dE_b}{d\mathbf{x}} \cdot \left( \frac{d^2 E_b}{d\mathbf{x}^2} \right)^{-1}$$

# Global spin rotation around the [100] axis I.



Néel

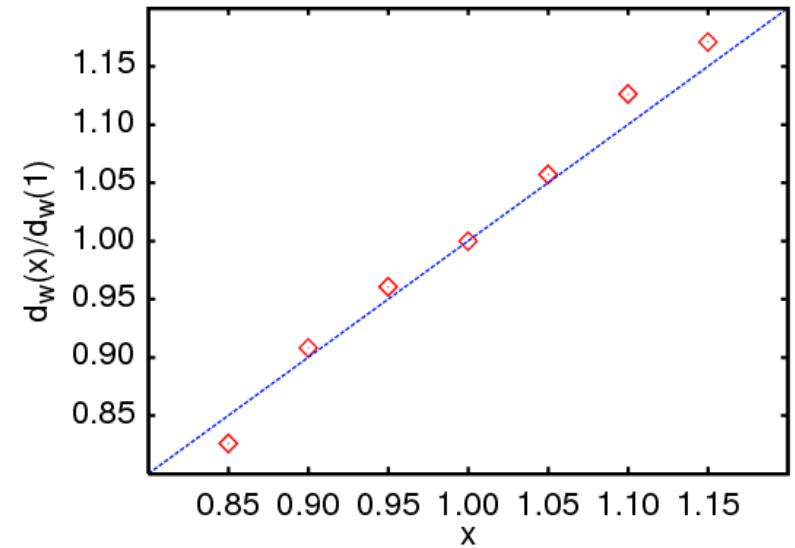
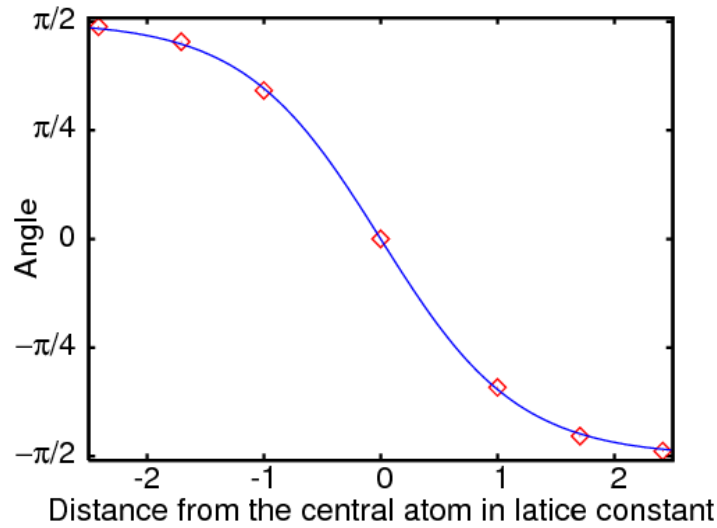
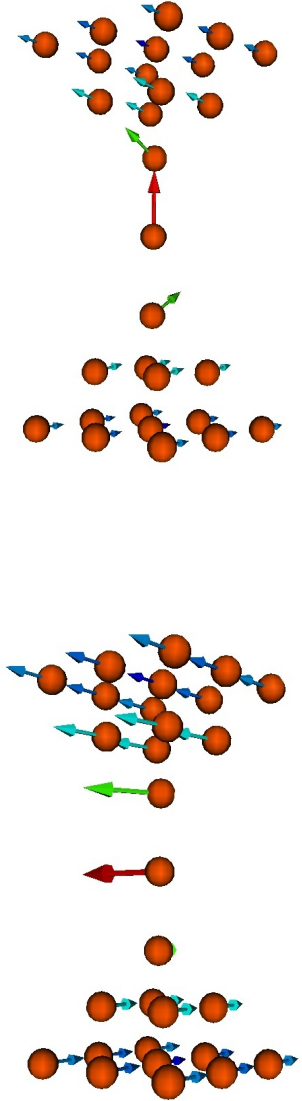


# Global spin rotation around the [100] axis II.

TABLE II. Fourier components of the band energy while rotating all spins around the [100] direction. The displayed coefficients are the  $a_k$  coefficients in the expansion  $E_b = a_0 + \sum_{k=1}^{\infty} a_k \cos(2k\theta)$ . The other coefficients are practically zeros. The units are Rydberg.

$x$	$k = 1$ ( $\times 10^{-3}$ Ry)	$k = 2$ ( $\times 10^{-4}$ Ry)	$k = 3$ ( $\times 10^{-5}$ Ry)	$k = 4$ ( $\times 10^{-5}$ Ry)	$k = 5$ ( $\times 10^{-5}$ Ry)
0.85	-0.466	0.108	2.920	2.658	2.623
0.90	-0.735	0.262	3.666	2.450	2.643
0.95	-1.003	1.029	2.190	2.407	2.812
1.00	-1.116	1.594	-0.296	3.459	2.370
1.05	-1.112	1.726	-0.897	3.823	2.237
1.10	-1.055	1.644	-0.613	3.718	2.303
1.15	-0.967	1.414	0.185	3.402	2.463

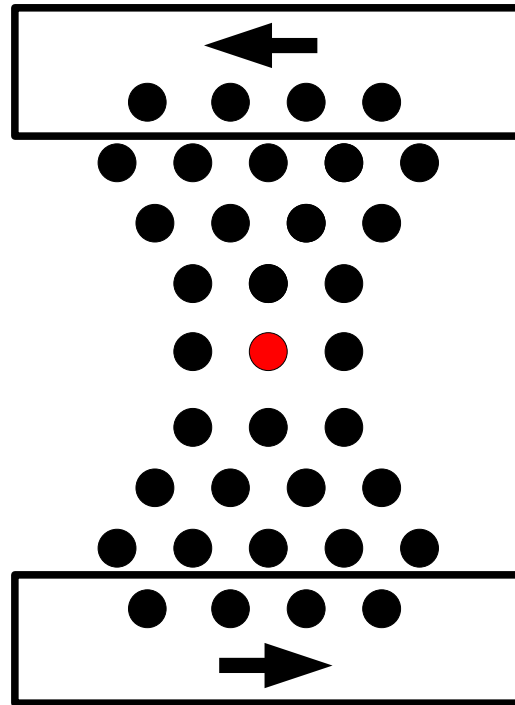
# Domain wall width



$$\varphi(z) = -\frac{\pi}{2} \text{th} \left( \frac{z}{2d_w} \right)$$

# Model of the energy function I.

$$E_{\text{band}}(\underbrace{\dots}_{\text{fixed}}, \vartheta, \varphi, \underbrace{\dots}_{\text{fixed}}) = \sum_{l=0}^{\infty} \sum_{m=-l}^l E_{lm} Y_{lm}(\vartheta, \varphi)$$



# Model of the energy function II.

$$E_{\text{band}}(\underbrace{\dots}_{\text{fixed}}, \vartheta, \varphi, \underbrace{\dots}_{\text{fixed}}) = \sum_{l=0}^{\infty} \sum_{m=-l}^l E_{lm} Y_{lm}(\vartheta, \varphi)$$

$l$	
1	$\frac{1}{2} \sqrt{\frac{3}{\pi}} z$
2	$\frac{1}{4} \sqrt{\frac{5}{\pi}} (3z^2 - 1)$
	$\frac{1}{4} \sqrt{\frac{15}{\pi}} (x^2 - y^2)$
3	$\frac{1}{4} \sqrt{\frac{7}{\pi}} (5z^3 - 3z)$
	$\frac{1}{4} \sqrt{\frac{105}{\pi}} (x^2 - y^2) z$
4	$\frac{3}{16} \sqrt{\frac{1}{\pi}} (35z^4 - 30z^2 + 3)$
	$\frac{3}{8} \sqrt{\frac{5}{\pi}} (x^2 - y^2) (7z^2 - 1)$
	$\frac{3}{16} \sqrt{\frac{35}{\pi}} (x^4 - 6x^2y^2 + y^4)$

$x = 1.00$
$-1.56 \cdot 10^{-2}$
$-2.38 \cdot 10^{-3}$
$3.75 \cdot 10^{-5}$
$5.22 \cdot 10^{-5}$
$7.91 \cdot 10^{-6}$
$3.63 \cdot 10^{-4}$
$7.96 \cdot 10^{-6}$
$-2.98 \cdot 10^{-6}$

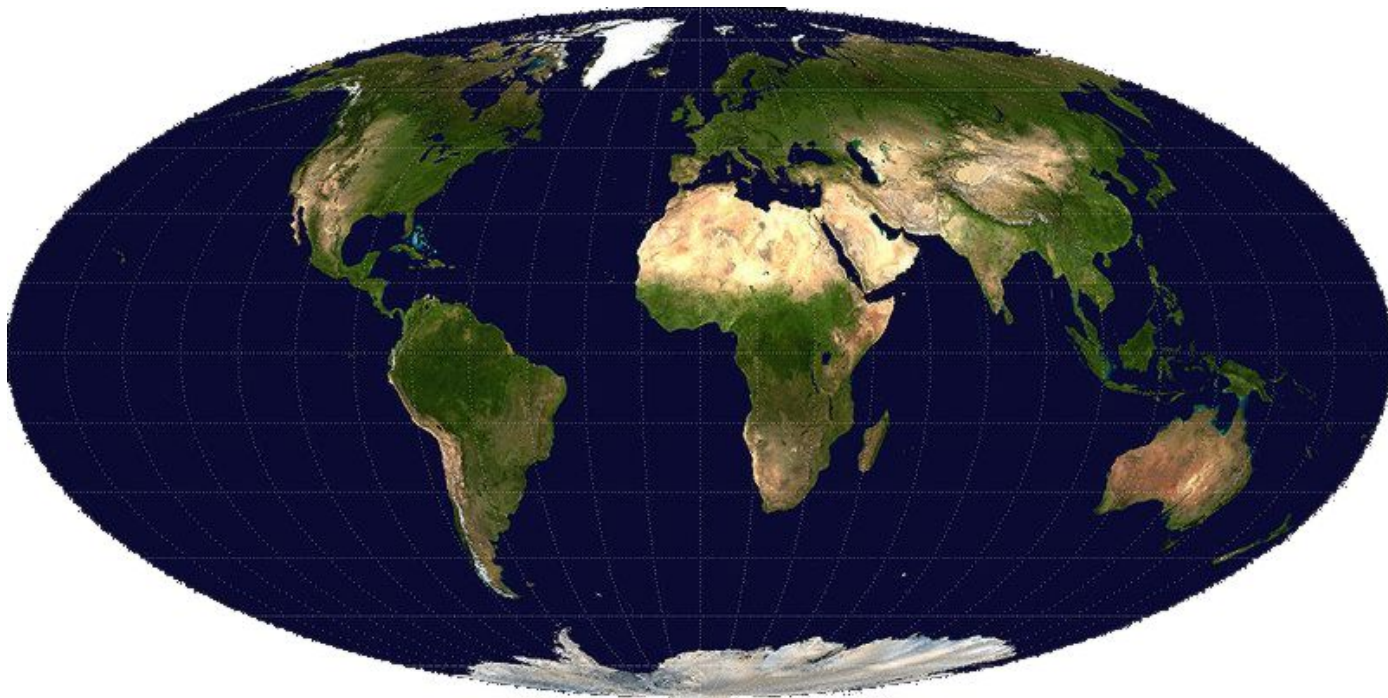
# Model of the energy function III.

$$E_{\text{band}}(\underbrace{\dots}_{\text{fixed}}, \vartheta, \varphi, \underbrace{\dots}_{\text{fixed}}) = \sum_{l=0}^{\infty} \sum_{m=-l}^l E_{lm} Y_{lm}(\vartheta, \varphi)$$

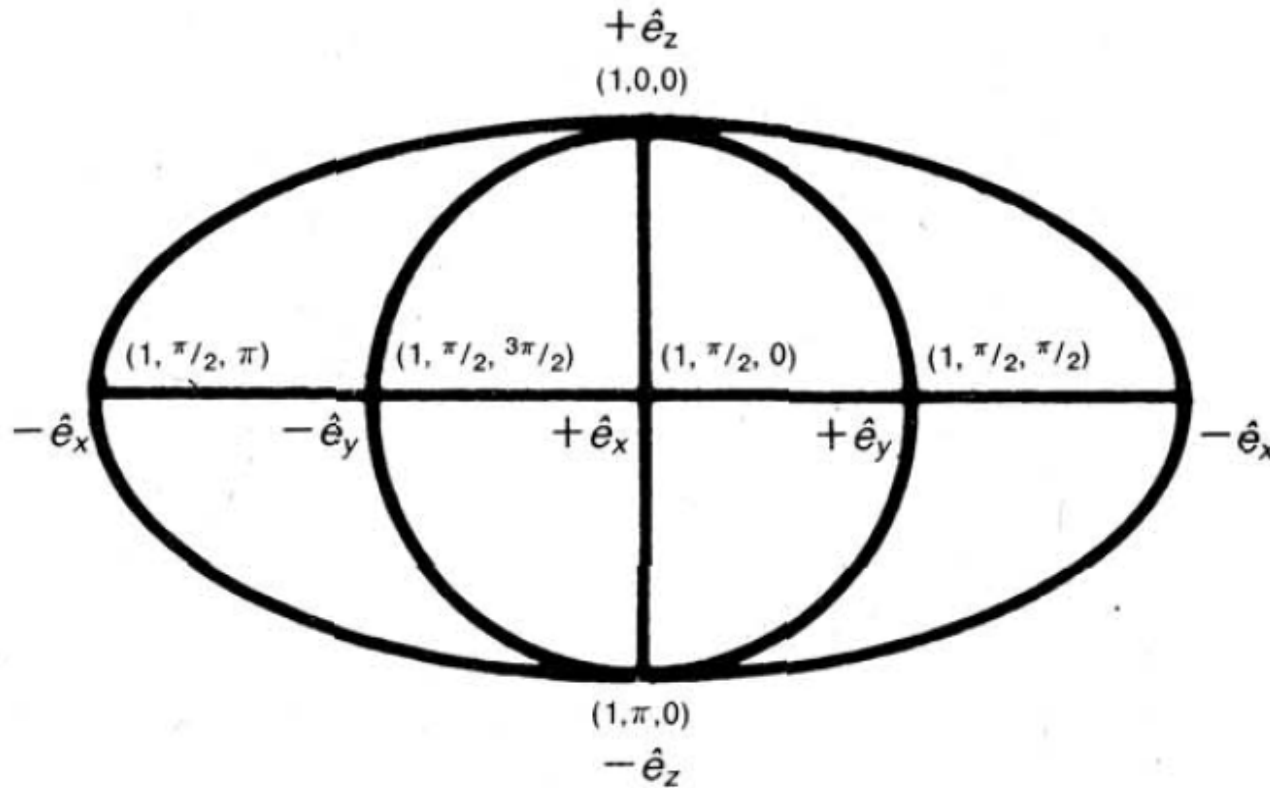
$l$		$x = 0.85$	$x = 0.90$	$x = 0.95$	$x = 1.00$	$x = 1.05$	$x = 1.10$	$x = 1.15$
1	$\frac{1}{2}\sqrt{\frac{3}{\pi}}z$	$-1.77 \cdot 10^{-2}$	$-1.81 \cdot 10^{-2}$	$-1.73 \cdot 10^{-2}$	$-1.56 \cdot 10^{-2}$	$-1.41 \cdot 10^{-2}$	$-1.29 \cdot 10^{-2}$	$-1.17 \cdot 10^{-2}$
2	$\frac{1}{4}\sqrt{\frac{5}{\pi}}(3z^2 - 1)$	$-1.86 \cdot 10^{-3}$	$-2.21 \cdot 10^{-3}$	$-2.44 \cdot 10^{-3}$	$-2.38 \cdot 10^{-3}$	$-2.27 \cdot 10^{-3}$	$-2.09 \cdot 10^{-3}$	$-1.88 \cdot 10^{-3}$
	$\frac{1}{4}\sqrt{\frac{15}{\pi}}(x^2 - y^2)$	$3.16 \cdot 10^{-4}$	$1.87 \cdot 10^{-4}$	$1.02 \cdot 10^{-4}$	$3.75 \cdot 10^{-5}$	$-2.13 \cdot 10^{-5}$	$-6.78 \cdot 10^{-5}$	$-1.00 \cdot 10^{-4}$
3	$\frac{1}{4}\sqrt{\frac{7}{\pi}}(5z^3 - 3z)$	$3.03 \cdot 10^{-4}$	$2.25 \cdot 10^{-4}$	$1.20 \cdot 10^{-4}$	$5.22 \cdot 10^{-5}$	$-2.03 \cdot 10^{-5}$	$-1.05 \cdot 10^{-4}$	$-1.96 \cdot 10^{-4}$
	$\frac{1}{4}\sqrt{\frac{105}{\pi}}(x^2 - y^2)z$	$-1.46 \cdot 10^{-5}$	$-6.82 \cdot 10^{-6}$	$3.17 \cdot 10^{-7}$	$7.91 \cdot 10^{-6}$	$1.44 \cdot 10^{-5}$	$1.96 \cdot 10^{-5}$	$2.16 \cdot 10^{-5}$
4	$\frac{3}{16}\sqrt{\frac{1}{\pi}}(35z^4 - 30z^2 + 3)$	$-4.60 \cdot 10^{-5}$	$1.26 \cdot 10^{-4}$	$3.38 \cdot 10^{-4}$	$3.63 \cdot 10^{-4}$	$3.71 \cdot 10^{-4}$	$3.57 \cdot 10^{-4}$	$3.17 \cdot 10^{-4}$
	$\frac{3}{8}\sqrt{\frac{5}{\pi}}(x^2 - y^2)(7z^2 - 1)$	$2.45 \cdot 10^{-6}$	$9.21 \cdot 10^{-6}$	$1.36 \cdot 10^{-5}$	$7.96 \cdot 10^{-6}$	$3.76 \cdot 10^{-6}$	$1.06 \cdot 10^{-7}$	$-3.83 \cdot 10^{-6}$
	$\frac{3}{16}\sqrt{\frac{35}{\pi}}(x^4 - 6x^2y^2 + y^4)$	$-5.05 \cdot 10^{-7}$	$-3.87 \cdot 10^{-7}$	$-1.36 \cdot 10^{-6}$	$-2.98 \cdot 10^{-6}$	$-6.45 \cdot 10^{-6}$	$-1.38 \cdot 10^{-5}$	$-2.54 \cdot 10^{-5}$

# Mollweide-projection

How to plot an  $E(\vartheta, \varphi)$  function?



# Mollweide-projection



$$x = 2\sqrt{2} \frac{\varphi}{\pi} \cos(\xi)$$

$$y = \sqrt{2} \sin(\xi)$$

$$2\xi + \sin(2\xi) = \pi \cos(\vartheta)$$

Figure 1. Mollweide's elliptical projection of the unit sphere based on the normal right-handed Cartesian coordinate system viewed towards the coordinate origin from along the  $+x$  axis.

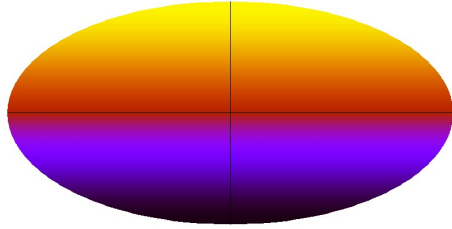
C. M. Quinn *et al.*, J. Chem. Edu., **61**, 569, (1984)

[http://en.wikipedia.org/wiki/Mollweide\\_projection](http://en.wikipedia.org/wiki/Mollweide_projection)

# Decomposition of the energy function [Ryd]

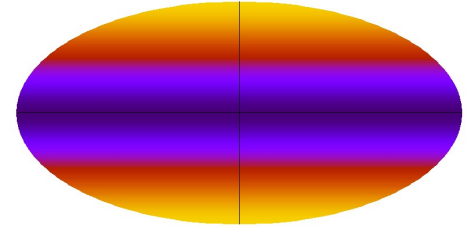
$l = 1$

$$-1.56 \cdot 10^{-2} \times$$



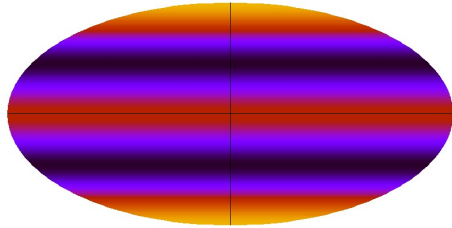
$l = 2$

$$-2.38 \cdot 10^{-3} \times$$

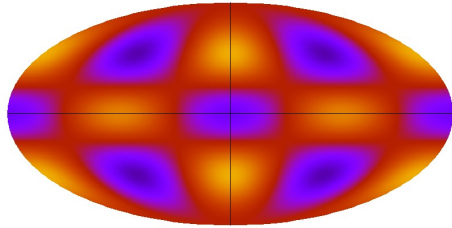


$l = 4$

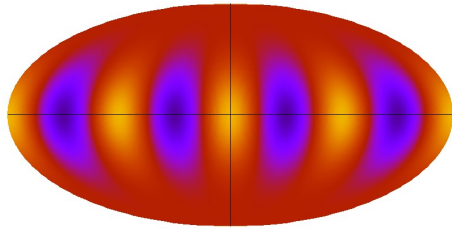
$$3.63 \cdot 10^{-4} \times$$



$$7.96 \cdot 10^{-6} \times$$

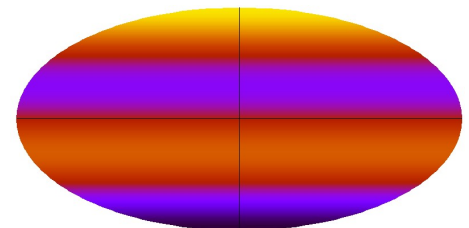


$$-2.98 \cdot 10^{-6} \times$$

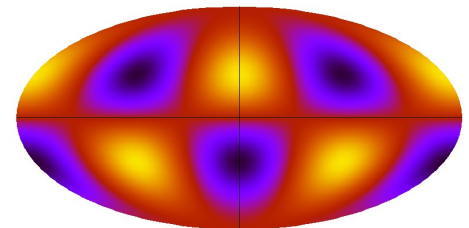


$l = 3$

$$5.22 \cdot 10^{-5} \times$$

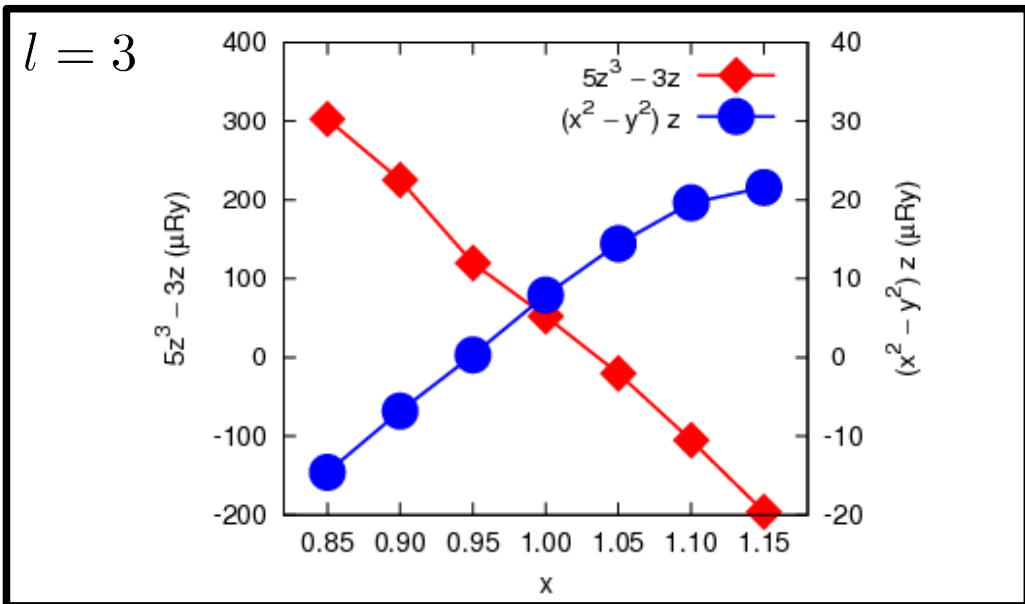
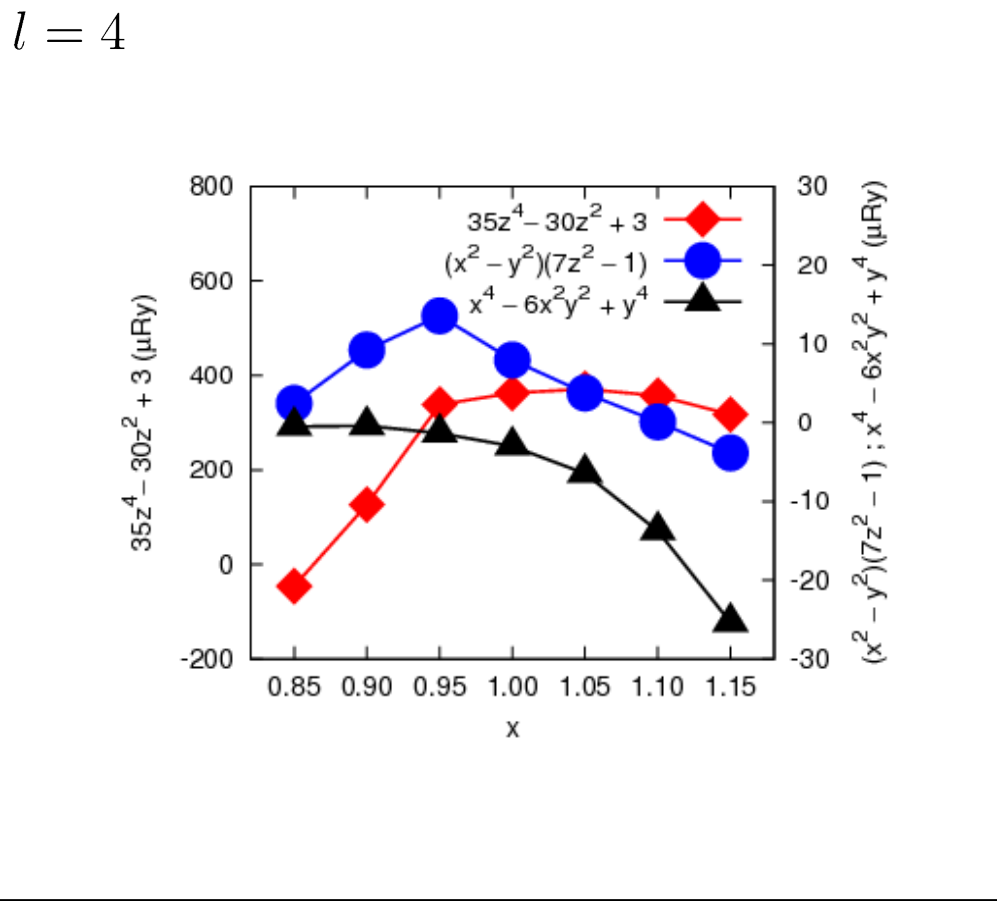
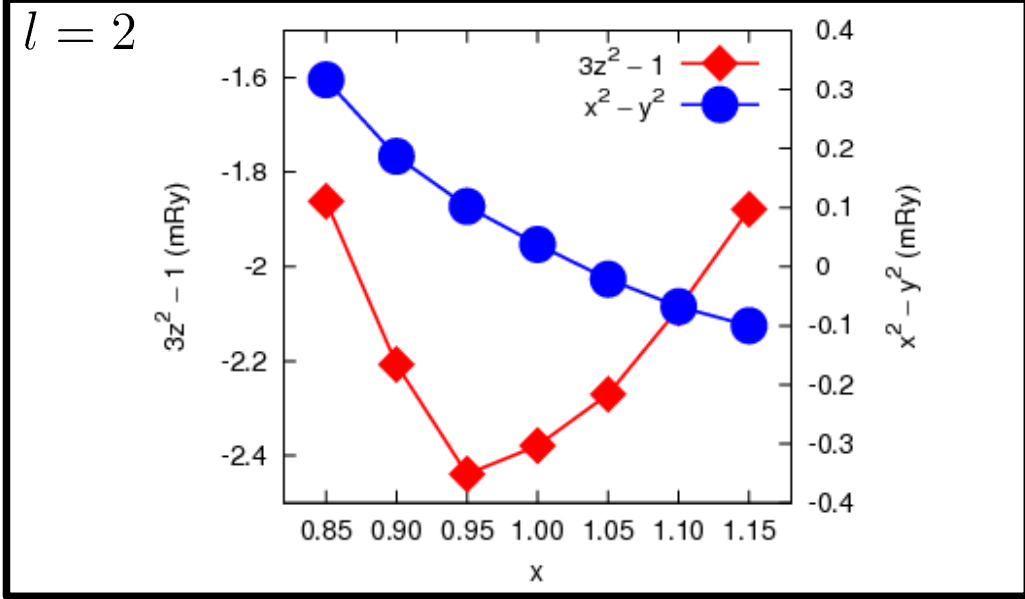
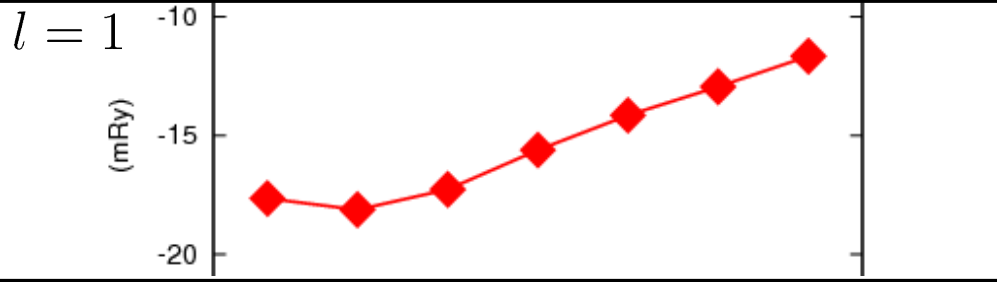


$$7.91 \cdot 10^{-6} \times$$





# Decomposition of the energy function II.

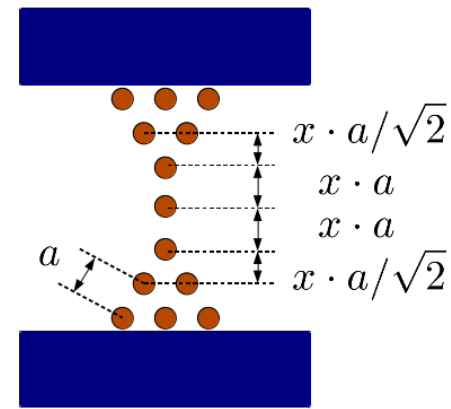


# Summary

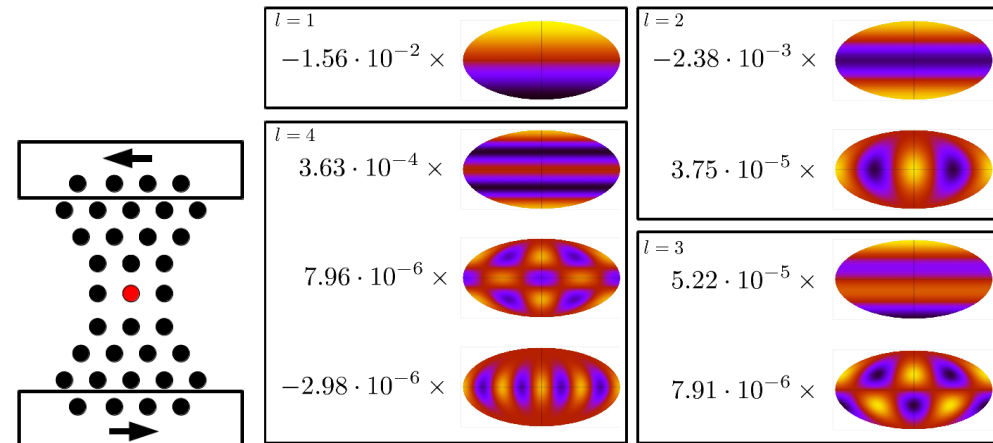
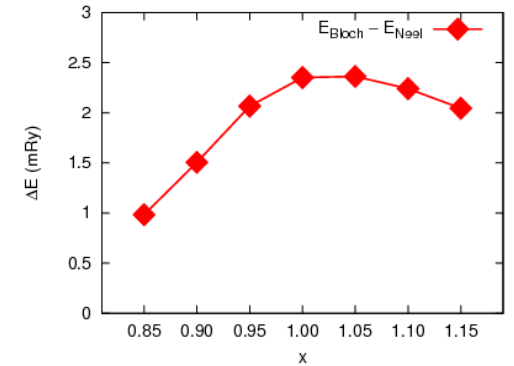
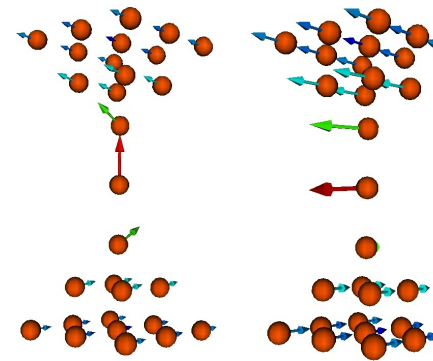
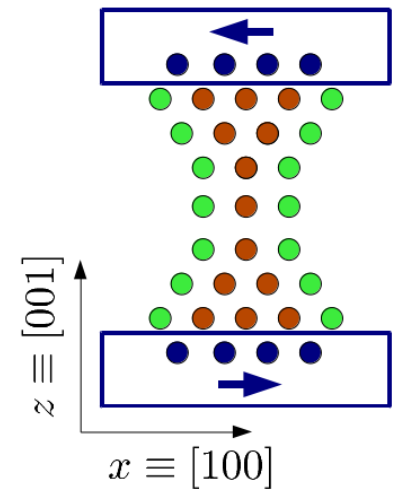
♣ A model of a cobalt nanocontact was constructed and investigated.

♣ The ground state was searched with multiscale approach: MC simulated annealing + NR optimization of the band energy.

♣ The Heisenberg model is not sufficient to map the magnetic energy landscape.



$$x \in \{0.85, \dots, 1.15\}$$



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# **Thank you for your attention**

**László Balogh, Krisztián Palotás, Bence Lazarovits,  
László Udvardi, László Szunyogh**

# Spin- and orbital moments on the atomic sites. (Néel str., $x = 1.00$ )

<i>3. layer</i>			<i>2. layer</i>		<i>1. layer</i>	<i>central atom</i>
1.85	1.78	1.85	1.91	1.91		
1.78	1.67	1.78			1.93	2.35
1.85	1.78	1.85	1.91	1.91		

<i>3. layer</i>			<i>2. layer</i>		<i>1. layer</i>	<i>central atom</i>
0.13	0.11	0.13	0.13	0.13		
0.10	0.07	0.10			0.16	1.71
0.13	0.11	0.13	0.13	0.13		

# Spin- and orbital moments on the central atom. (Néel str.)

