Simulations in Statistical Physics

Course for MSc physics students

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Subjects

- Self-Organized Criticality
- Bak-Sneppen model of evolution
- Traffic models
- 1d driven systems

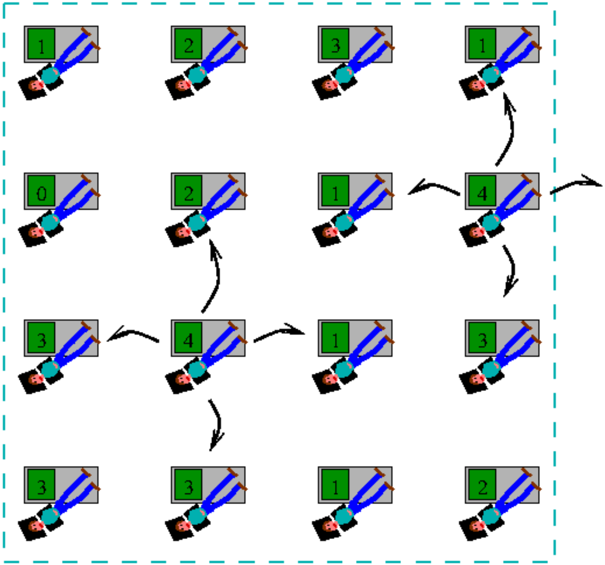
Self-Organized Criticality

- Critical state: inflection point in the critical isotherm
- Power law functions of correlation length, ralaxation time
- Control parameter, generally temperature
- Critical point as an attractor?
- Why? Power law: We see many cases
  - $1/f$ noise (music, ocean, earthquakes, flames)
  - Lack of scales (market, earthquakes)
- Underlying mechanism?

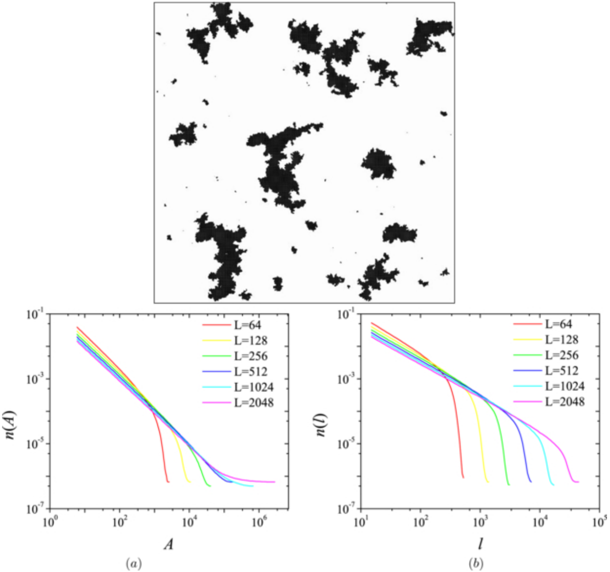
Bak-Tang-Wiesenfeld model

- Originally a sandpile model
- Better explained as a *Lazy Bureaucrat model*:
  - Bureaucrats are sitting in a large office in a square lattice arrangement
  - Occasionally the boss comes with a dossier and places it on a random table
  - The bureaucrats do *nothing* until they have less than 4 dossiers on their table
  - Once a bureaucrat has 4 or more dossiers on its table starts to panic and distributes its dossiers to its 4 neighbors
  - The ones sitting at the windows give also 1 dossier to its neighbors and throw the rest out of the window.

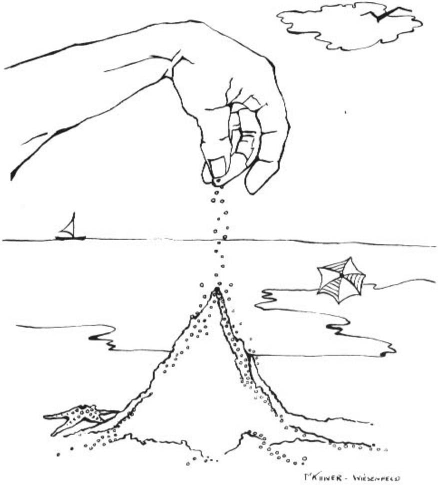
Bak-Tang-Wiesenfeld model



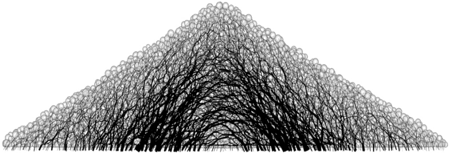
Bak-Tang-Wiesenfeld results



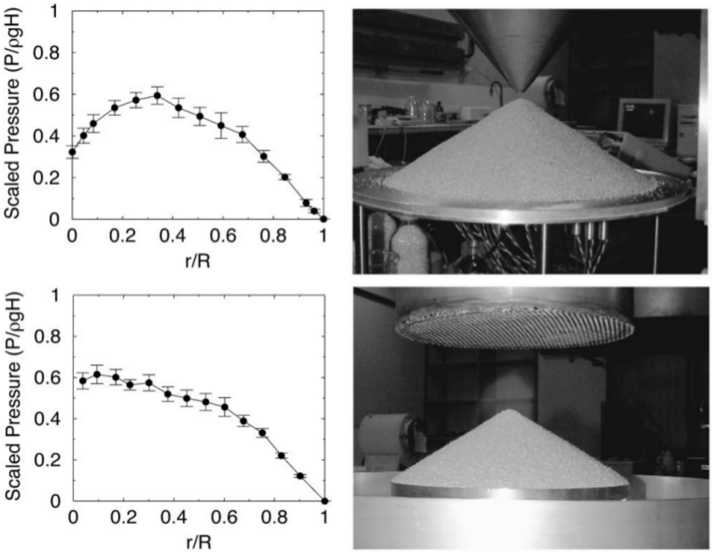
Sandpile experiment



Dip under the heap



Dip under the heap



Forest fire

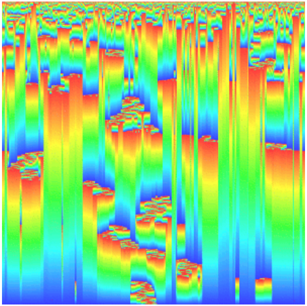
Forest fire model

- Burning cell turns into an empty cell
- A tree will burn if at least one neighbor is burning
- A tree ignites with probability f even if no neighbor is burning
- An empty space fills with a tree with probability p

- Control parameter p/f the average number of trees planted between two lightning strikes
- Histogram of burned forest size is a power law

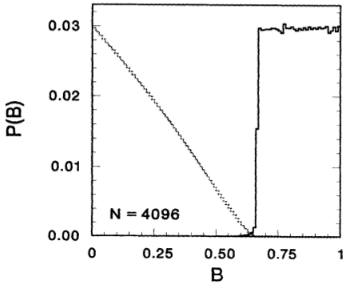
Bak-Sneppen model of evolution

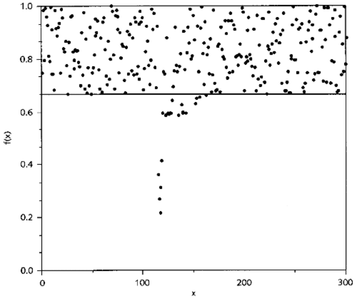
- N species all depends on two other (ring geometry)
- Each species are characterized by a single *fitness*
- In each turn the species with the lowest fitness dies out and with it its two neighbors irrespective of their fitness
- These 3 species are replaced by new ones with random fitness
- Inital and update fitness is uniform between $[0, 1]$

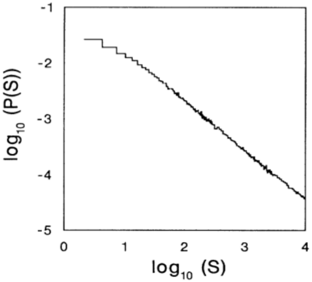


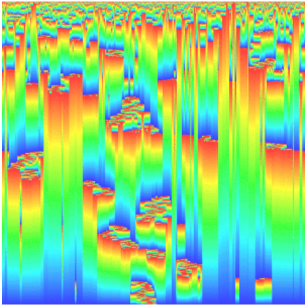
Bak-Sneppen model of evolution: Results

- Steady state with avalanches
- Avalanches start with a fitness $f > f_c \simeq 0.66$
- Avalanche size distribution power law
- Distance correlation power law

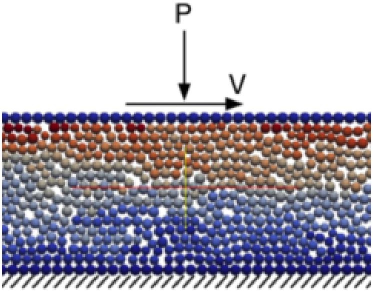

P(B)
0.03
0.02
0.01
0.00
N = 4096
0
0.25
0.50
0.75
1
B

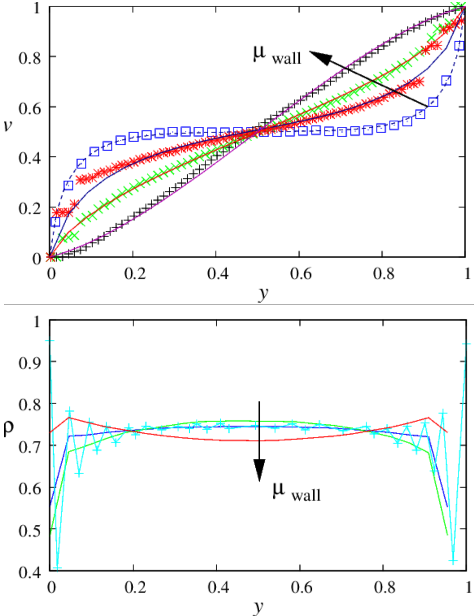

1.0
0.8
0.6
0.4
0.2
0.0
f(x)
0
100
200
300
x


log10 (P(S))
-1
-2
-3
-4
-5
0
1
2
3
4
log10 (S)



Bak-Sneppen model of evolution an application: Granular shear



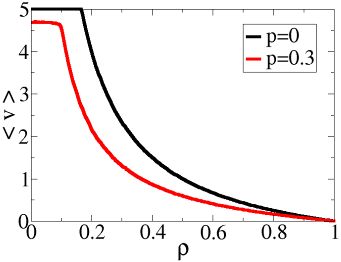


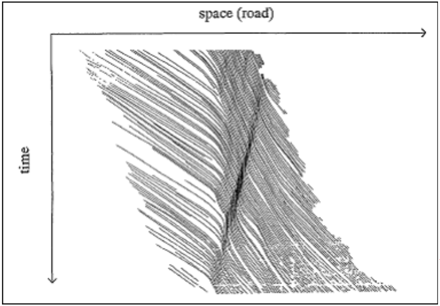
Traffic models

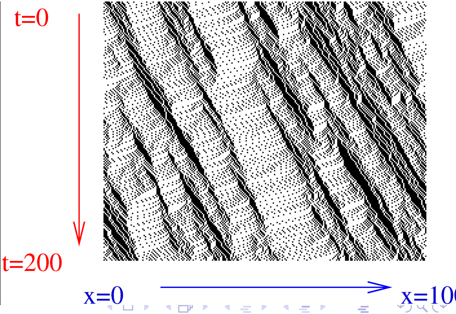
Nagel–Schreckenberg model

- Periodic 1d lattice (ring) Autobahn
- Cars occupying a lattice moving with velocities $0, 1, 2, 3, 4, 5$
- Remark, if max speed is 126 km/h, then lattice length is 7 m, a very good guess for a car in a traffic jam
- It uses parallel update
- Simultaneously each car adjusts its speed according to rules:
  1. **Acceleration**: All cars not at the maximum velocity increase their velocity by 1
  2. **Slowing down**: Speed is reduced to distance ahead (1 sec rule)
  3. **Randomization**: With probability p speed is reduced by 1
  4. **Car motion**: Each car moves forward the number of cells equal to their velocity.

Emergence of traffic jams







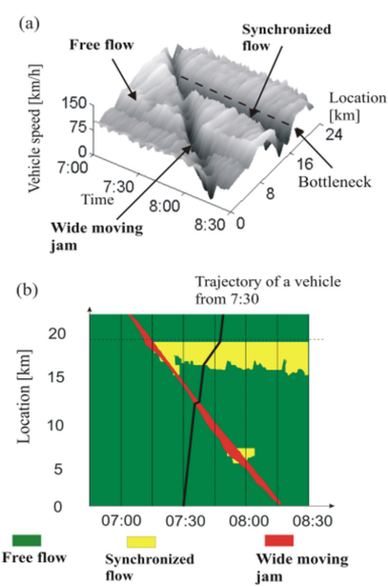
Nagel–Schreckenberg model

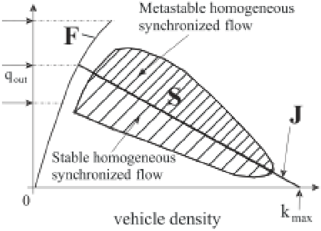
- Transition from free-flow to jammed state
- Jammed state is a phase-separated phase
- Without randomization a sharp transition
- Used in NRW to predict traffic jams

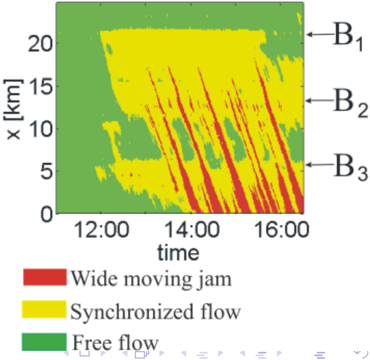

autobahn.NRW
Hochrechnung für Bundesautobahnen
Montag, 01.07.2013, 08:05 h
aktuelle Verkehrslage
Prognose 30 min
Prognose 60 min
+7
Langzeit Prognose
Fahrzeitprognose
Farbmodus: Standard
Baustelleninfo
Verkehrsdichten
frei
zähfließend
dicht
gestaut
Baustelle mit geringer Staugefahr
Baustelle mit hoher Staugefahr
Anschlussstelle
Sperrung der Autobahn
Vollsperrung der Anschlussstelle
Teilsperrung der Anschlussstelle
Informationen
Impressum
Version 3.6.1
0 km
15 km
Wesel
Recklinghausen
Lünen
Geldern
Kamp-Lintfort
Bottrop
Gelsenk.
Herne
Dortmund
Moers
Oberhsn.
Essen
Bochum
Duisburg
Mülheim
Witten
Kempen
Krefeld
Hagen
Velbert
Gevelsberg
Viersen
Düsseldorf
Mönchengladbach
Neuss
Wuppertal
Lüdenscheid

Three-phase traffic theory

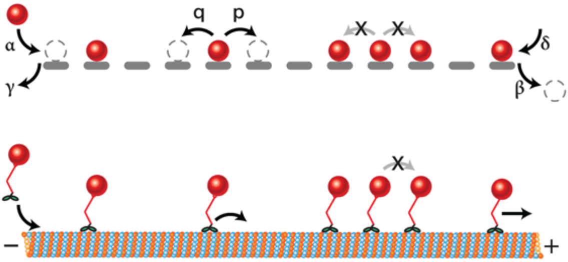
Three traffic phases





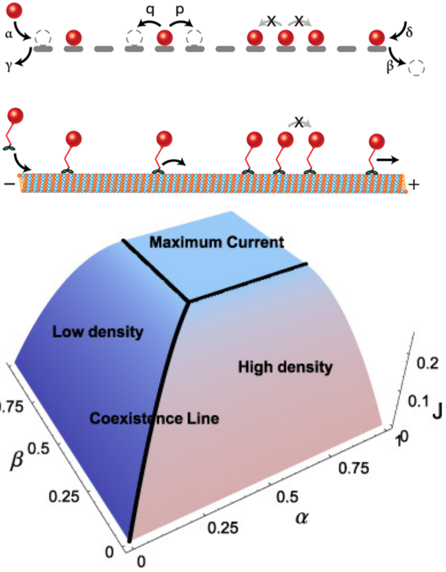


Asymmetric simple exclusion process



- $p + q = 1$
- If $p = q$ then SEP a Markov-process
- Generally $\gamma = \delta = 0$
- α and β determines the phase diagram

Asymmetric simple exclusion process



Three state ASEP

$$+\ 0 \xrightarrow{\alpha} 0\ + \quad ; \quad 0\ - \xrightarrow{\alpha} -\ 0 \quad ; \quad +\ - \underset{q}{\overset{1}{\rightleftarrows}} -\ +$$

- If q small three blocks $(00\dots00++\cdots++--\cdots--)$
- Mixed state above $q=1$
- Numerical simulations suggested an other phase transition at $q_c < 1$
- Actually false, only correlation length is finite but large $\sim \mathcal{O}(10^{70})$
- Correspondance to Zero Range Process