Vector Formulas
1 oldal

1 $\mathbf{a}\cdot(\mathbf{b}\times\mathbf{c}) = \mathbf{b}\cdot(\mathbf{c}\times\mathbf{a}) = \mathbf{c}\cdot(\mathbf{a}\times\mathbf{b})$

2 $\mathbf{a}\times(\mathbf{b}\times\mathbf{c}) = (\mathbf{a}\cdot\mathbf{c})\mathbf{b} - (\mathbf{a}\cdot\mathbf{b})\mathbf{c}$

3 $(\mathbf{a}\times\mathbf{b})\cdot(\mathbf{c}\times\mathbf{d}) = (\mathbf{a}\cdot\mathbf{c})(\mathbf{b}\cdot\mathbf{d}) - (\mathbf{a}\cdot\mathbf{d})(\mathbf{b}\cdot\mathbf{c})$

4 $\nabla\times\nabla\psi = 0$

5 $\nabla\cdot(\nabla\times\mathbf{a}) = 0$

6 $\nabla\times(\nabla\times\mathbf{a}) = \nabla(\nabla\cdot\mathbf{a}) - \nabla^2\mathbf{a}$

7 $\nabla\cdot(\psi\mathbf{a}) = \mathbf{a}\cdot\nabla\psi + \psi\nabla\cdot\mathbf{a}$

8 $\nabla\times(\psi\mathbf{a}) = \nabla\psi\times\mathbf{a} + \psi\nabla\times\mathbf{a}$

9 $\nabla(\mathbf{a}\cdot\mathbf{b}) = (\mathbf{a}\cdot\nabla)\mathbf{b} + (\mathbf{b}\cdot\nabla)\mathbf{a} + \mathbf{a}\times(\nabla\times\mathbf{b}) + \mathbf{b}\times(\nabla\times\mathbf{a})$

10 $\nabla\cdot(\mathbf{a}\times\mathbf{b}) = \mathbf{b}\cdot(\nabla\times\mathbf{a}) - \mathbf{a}\cdot(\nabla\times\mathbf{b})$

11 $\nabla\times(\mathbf{a}\times\mathbf{b}) = \mathbf{a}(\nabla\cdot\mathbf{b}) - \mathbf{b}(\nabla\cdot\mathbf{a}) + (\mathbf{b}\cdot\nabla)\mathbf{a} - (\mathbf{a}\cdot\nabla)\mathbf{b}$

If $\mathbf{x}$ is the coordinate of a point with respect to some origin, with magnitude $r = |\mathbf{x}|$, $\mathbf{n} = \mathbf{x}/r$ is a unit radial vector, and $f(r)$ is a well-behaved function of r, then

12 $\nabla\cdot\mathbf{x} = 3 \qquad \nabla\times\mathbf{x} = 0$

13 $\nabla\cdot[\mathbf{n}f(r)] = \frac{2}{r}f + \frac{\partial f}{\partial r} \qquad \nabla\times[\mathbf{n}f(r)] = 0$

14 $(\mathbf{a}\cdot\nabla)\mathbf{n}f(r) = \frac{f(r)}{r}[\mathbf{a} - \mathbf{n}(\mathbf{a}\cdot\mathbf{n})] + \mathbf{n}(\mathbf{a}\cdot\mathbf{n})\frac{\partial f}{\partial r}$

$$\nabla(\mathbf{x}\cdot\mathbf{a}) = \mathbf{a} + \mathbf{x}(\nabla\cdot\mathbf{a}) + i(\mathbf{L}\times\mathbf{a})$$

where $\mathbf{L} = \frac{1}{i}(\mathbf{x}\times\nabla)$ is the angular-momentum operator.

Theorems from Vector Calculus

2.oldal

In the following ϕ, ψ, and $\mathbf{A}$ are well-behaved scalar or vector functions, V is a three-dimensional volume with volume element d^3x, S is a closed two-dimensional surface bounding V, with area element da and unit outward normal $\mathbf{n}$ at da.

$\int_V \nabla\cdot\mathbf{A}\, d^3x = \int_S \mathbf{A}\cdot\mathbf{n}\, da$ (Divergence theorem) 1

$\int_V \nabla\psi\, d^3x = \int_S \psi\mathbf{n}\, da$ 2

$\int_V \nabla\times\mathbf{A}\, d^3x = \int_S \mathbf{n}\times\mathbf{A}\, da$ 3

$\int_V (\phi\nabla^2\psi + \nabla\phi\cdot\nabla\psi)\, d^3x = \int_S \phi\mathbf{n}\cdot\nabla\psi\, da$ (Green's first identity) 4

$\int_V (\phi\nabla^2\psi - \psi\nabla^2\phi)\, d^3x = \int_S (\phi\nabla\psi - \psi\nabla\phi)\cdot\mathbf{n}\, da$ (Green's theorem) 5

In the following S is an open surface and C is the contour bounding it, with line element $d\mathbf{l}$. The normal $\mathbf{n}$ to S is defined by the right-hand-screw rule in relation to the sense of the line integral around C.

$\int_S (\nabla\times\mathbf{A})\cdot\mathbf{n}\, da = \oint_C \mathbf{A}\cdot d\mathbf{l}$ (Stokes's theorem) 6

$\int_S \mathbf{n}\times\nabla\psi\, da = \oint_C \psi\, d\mathbf{l}$ 7

A témához kapcsolódó fontosabb összefüggések :

$Ar^n + B\frac{1}{r^{n+1}}$, $P_n^m(\cos\varphi)$, $\frac{3x^2-1}{2}$,

$Ae^{kz} + Be^{-kz}$, $Ash(kz) + Bch(kz)$, $Az + B$,

$A\sin(n\varphi) + B\cos(n\varphi)$, $A\varphi + B$,

$AJ_n(kr) + BN_n(kr)$, $Ar^n + B\frac{1}{r^n}$, $A + B\ln(r)$