The Schrödinger equation describes a single particle. The square of the wave function for a particle must then describe the *probability density,* which is the probability per unit volume, of finding the particle at a location. The probability of finding the particle in some volume element must also be proportional to the size of the volume element dV. Thus, in one dimension, the probability of finding a particle in a region dx at the position x is $\psi^2(x)\,dx$. If we call this probability $P(x)\,dx$, where $P(x)$ is the **probability density,** we have

$$P(x) = \psi^2(x) \tag{34-17}$$

PROBABILITY DENSITY

Generally the wave function depends on time as well as position, and is written $\psi(x,t)$. However, for standing waves, the probability density is independent of time. Since we will be concerned mostly with standing waves in this chapter, we omit the time dependence of the wave function and write it $\psi(x)$ or just ψ.

The probability of finding the particle in dx at point x_1 or at point x_2 is the sum of the separate probabilities $P(x_1)\,dx + P(x_2)\,dx$. If we have a particle at all, the probability of finding the particle somewhere must be 1. Then the sum of the probabilities over all the possible values of x must equal 1. That is,

$$\int_{-\infty}^{\infty} \psi^2\,dx = 1 \qquad 34\text{-}18$$

NORMALIZATION CONDITION

Equation 34-18 is called the **normalization condition.** If ψ is to satisfy the normalization condition, it must approach zero as x approaches infinity. This places a restriction on the possible solutions of the Schrödinger equation. There are solutions to the Schrödinger equation that do not approach zero as x approaches infinity. However, these are not acceptable as wave functions.