

# Stability of time-reversal symmetry breaking spin liquid states in high-spin fermionic systems

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- Part I

- Fundamentals of high spin systems
- Spin wave description
- Valence bond picture

- Part II

- Competing spin liquid states of spin-3/2 fermions in a square lattice
- Competing spin liquid states of spin-5/2 fermions in a honeycomb lattice
  - Properties of chiral spin liquid state
  - Stability of the spin liquid states beyond the mean-field approximation
  - Finite temperature behavior
  - Experimentally measurable quantities

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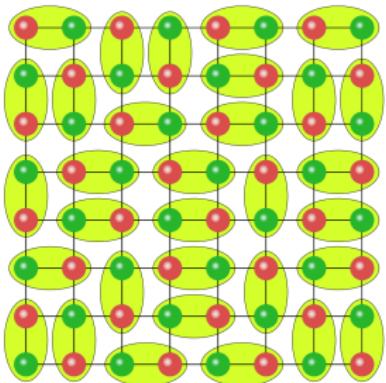
PRA 88(R) 043619 (2013)

PRA 84 011611 (2011)

EPL 93, 66005 (2011)

## Part II

# Spin liquid phases



**Bond operators**

$$\chi_{i,j} = c_{i,\sigma}^\dagger c_{j,\sigma} = \chi_{j,i}^\dagger$$

$$H = J \sum_{\langle i,j \rangle} \vec{S}_i \cdot \vec{S}_j$$

$J > 0$  antiferromagnetic coupling

**Schwinger fermions**

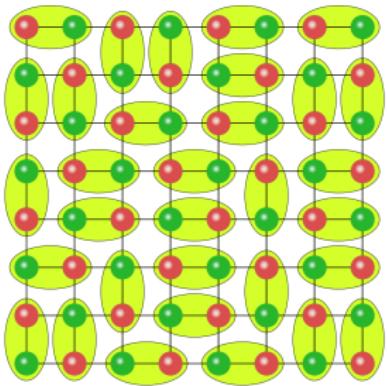
$$\vec{S}_i = \sum_{\alpha,\beta} c_{i,\alpha}^\dagger \vec{F}_{\alpha,\beta} c_{i,\beta}$$

$$\{c_{i,\alpha}, c_{j,\beta}^\dagger\} = \delta_{i,j} \delta_{\alpha,\beta}$$

1 particle / site

$$H = -J \sum_{\langle i,j \rangle} c_{i,\alpha}^\dagger c_{j,\alpha} c_{j,\beta}^\dagger c_{i,\beta} + \sum_i \varphi_i (c_{i,\alpha}^\dagger c_{i,\alpha} - 1)$$

# Spin liquid phases



## Bond operators

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## Local gauge invariance

$$c_{j,\alpha} \rightarrow e^{i\theta_j} c_{j,\alpha}$$

$$c_{j,\alpha}^\dagger \rightarrow e^{-i\theta_j} c_{j,\alpha}^\dagger$$

$$\chi_{i,j} \rightarrow \chi_{i,j} e^{i(\theta_j - \theta_i)}$$

$$\varphi_i \rightarrow \varphi_i - i\partial_t \varphi_i$$

## Plaquette operator

$$\Pi = \chi_{i,j} \chi_{j,k} \cdots \chi_{l,i}$$

Mean-field bonds:  $\chi_{i,j} \rightarrow \bar{\chi}_{i,j}$

## Plaquette operator

$$\Pi = \bar{\chi}_{i,j} \bar{\chi}_{j,k} \dots \bar{\chi}_{l,i} = \text{Abs}[\Pi] e^{i\Phi}$$

- $\Phi = n\pi$  ( $n$  integer): no time reversal symmetry breaking.
- With nontrivial  $\Phi$ : time reversal symmetry breaking.
  - Low lying excitations composite particle  
(spinon + an elementary flux)  $\rightarrow$  **anyons**.
  - Effective gauge theory.

$F = \frac{3}{2}$  system on a square lattice

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Let us recall the n.n. Hamiltonian in the limit of strong repulsion:

$$H_{\text{eff}}^{F=3/2} = \sum_{\langle i,j \rangle} \left[ g_0 \mathcal{P}_0^{(i,j)} + g_2 \mathcal{P}_2^{(i,j)} \right]$$

where  $\mathcal{P}_S^{(i,j)} = c_{i,\sigma_1}^\dagger c_{j,\sigma_2}^\dagger c_{j,\sigma_3} c_{i,\sigma_4} \hat{P}_S$ , and  $\hat{P}_S$  are antisymmetric.

Hamiltonian with 2 parameters.

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Hamiltonian with 2 parameters.

$$H_{\text{eff}}^{F=3/2} = \sum_{\langle i,j \rangle} \left[ a_0 n_i n_j + a_1 \mathbf{S}_i \cdot \mathbf{S}_j + a_2 (\mathbf{S}_i \cdot \mathbf{S}_j)^2 + a_3 (\mathbf{S}_i \cdot \mathbf{S}_j)^3 \right]$$

Hamiltonian with 4 parameters

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Hamiltonian with 4 parameters

One can exploit the fact that  $\hat{P}_0$  and  $\hat{P}_2$  are antisymmetric with respect to the exchange of the spin of the colliding particles leading to a new effective Hamiltonian:

$F = \frac{3}{2}$  system on square lattice

$$H_{\text{eff}} = \sum_{\langle i,j \rangle} \left[ g_0 \mathcal{P}_0^{(i,j)} + g_2 \mathcal{P}_2^{(i,j)} \right]$$

$$H_{\text{eff}} = \sum_{\langle i,j \rangle} \left[ a_n \left( n_i n_j + \chi_{i,j}^\dagger \chi_{i,j} - n_i \right) + a_s \left( \mathbf{S}_i \mathbf{S}_j + \mathbf{B}_{i,j}^\dagger \mathbf{B}_{i,j} - \frac{15}{4} n_i \right) \right]$$

E. Sz. and M. Lewenstein EPL 93 66005 (2011)

## Site- and bond-centered operators

- $n_i = c_{i,\alpha}^\dagger c_{i,\alpha}$  (particle number at site  $i$ )
- $\mathbf{S}_i = c_{i,\alpha}^\dagger \mathbf{F}_{\alpha,\beta} c_{i,\beta}$  (spin at site  $i$ )
- $\chi_{i,j} = c_{i,\alpha}^\dagger c_{j,\alpha}$  (scalar valence bonds)
- $\mathbf{B}_{i,j} = c_{i,\alpha}^\dagger \mathbf{F}_{\alpha,\beta} c_{j,\beta}$  (vector valence bonds)

## Nonuniform bond centered orders

- $\langle |\chi_{i,j}|^2 \rangle \propto \langle \mathbf{S}_i \mathbf{S}_j \rangle \rightarrow$  spin-Peierls distortion      B. Marston and J. Affleck PRB (1989)
- $\langle |\mathbf{B}_{i,j}|^2 \rangle \propto \langle \mathbf{Q}_i \mathbf{Q}_j \rangle \rightarrow$  quadrupole-Peierls distortion

# SU(2) bond-ordered states

The nonlocal part of the mean-field Hamiltonian:

$$\left( a_n \langle \chi_{j,i} \rangle \delta_{\alpha,\beta} + a_s \langle \mathbf{B}_{j,i} \rangle \mathbf{F}_{\alpha,\beta} \right) c_{i,\alpha}^\dagger c_{j,\beta} + H.c.$$

- A more suitable new link parameter:  $U_{i,j} = \langle \mathbf{B}_{i,j} \rangle \mathbf{F}$ , with the usual inner product in the 3 dimensional space of the generators  $\mathbf{F}$ .
  - $U_{i,j}$  is a member of SU(2)
  - $4 \times 4$  matrix for  $F = 3/2$  fermionic atoms
- The SU(2) plaquette:  $\Pi^{SU(2)} = U_{i,j} U_{j,k} U_{k,l} U_{l,i}$ .
- The SU(2) plaquette  $\Pi^{SU(2)}$  is also invariant under the U(1) gauge transformation defined above:  $c_{i,\sigma} \rightarrow c_{i,\sigma} e^{i\phi_i}$ ,  $\langle \chi_{i,j} \rangle \rightarrow \langle \chi_{i,j} \rangle e^{i(\phi_j - \phi_i)}$ , and  $U_{i,j} \rightarrow U_{i,j} e^{i(\phi_j - \phi_i)}$ .

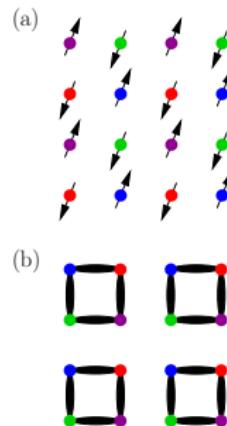
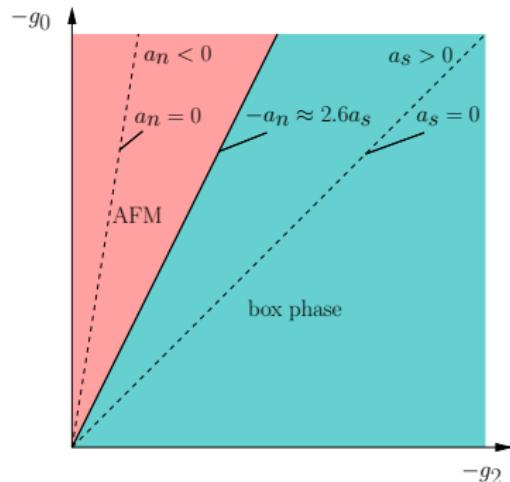
# Mean-field phase diagram: $F = 3/2, 1/4$ filling, square lattice

## Parameters

$$a_n = \frac{5g_2 - g_0}{4}$$

$$a_s = \frac{(g_2 - g_0)}{3}$$

$$g_S = -\frac{4t}{U_S}$$



## Nonzero order parameters

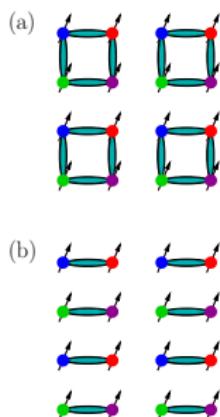
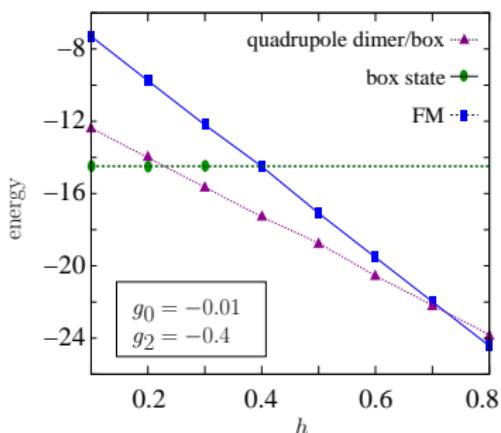
- AFM:  $\langle \mathbf{S}_i \rangle$
- box phase:  $\langle \chi_{i,j} \rangle$ 
  - valence bond solid state
  - plaquettes with flux  $\Phi = 0$ , or  $\pm\pi$

For the SU(4) line: C. Wu MPL (2006)  
E. Sz. and M. Lewenstein EPL 93 66005 (2011)

# In presence of magnetic field

Hamiltonian with magnetic field:

$$H^h = H^{MF} + h \sum_i \mathbf{S}_i$$



## Order parameters

- SU(2) dimer/  
Quadrupole dimer:  
 $\langle \mathbf{S}_i \rangle, \langle \chi_{i,j} \rangle, \langle \mathbf{B}_{i,j} \rangle$
- SU(2) plaquette/  
Quadrupole plaquette:  
 $\langle \mathbf{S}_i \rangle, \langle \chi_{i,j} \rangle, \langle \mathbf{B}_{i,j} \rangle$

# In presence of magnetic field

Linear Zeeman energy:  $\omega_L = g_F \mu_B B$

Quadratic Zeeman energy:  $\omega_q = \frac{\omega_L^2}{\omega_{hf}}$ .

- $\omega_{hf}$ : hyperfine splitting (1-10 GHz)
- $g_F$ : gyromagnetic factor
- $\mu_B$ : Bohr magneton

The quadratic  
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Magnetic field was considered in units of  $t$ .

$t$  has a maximum at

$V_0 \approx \omega_R$ ,

where  $t \sim \omega_R \sim 1 - 100$  kHz.

( $\omega_R / 2\pi \sim 400.98$  kHz for  ${}^9\text{Be}$ )

In optical lattice:  $t = \omega_R \frac{2}{\pi} \xi^3 e^{-2\xi^2}$

- $\xi = (V_0 / \omega_R)^{1/4}$
- $V_0$  potential depth,
- $\omega_R$  recoil energy

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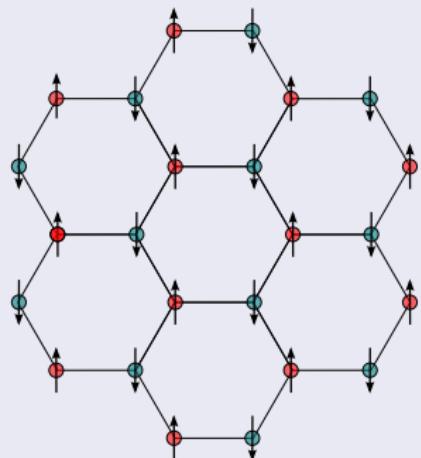
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$$\omega_{hf} / \omega_L \sim 10^4 - 10^7$$

# Competing spin liquid states of SU(6) symmetric spin-5/2 fermions on a honeycomb lattice

# Néel state vs. spin liquid state

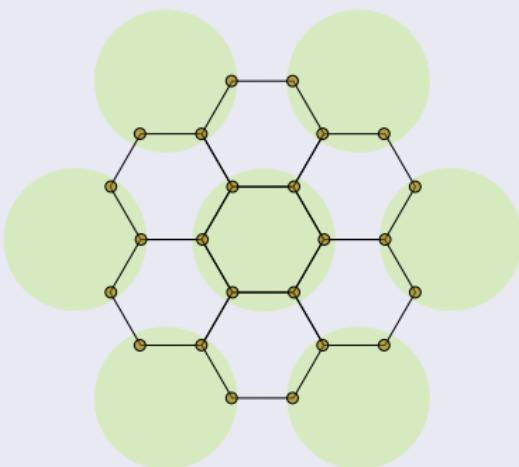
## Néel state



Néel state is not energetically favorable.

M. Hermele, V. Gurarie, A. M. Rey, PRL (2009).

## Spin liquid - VB state



The ground state expectedly consists SU(6) singlets  
(6 atoms with  $F = \frac{5}{2}$  can form an SU(6) singlet)

## Finite temperature field theory

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Decoupling procedure:

Hubbard-Stratonovich transformation:  $L[c, \bar{c}] \rightarrow L[c, \bar{c}; \varphi, \chi]$

auxiliary fields:  $\varphi_i$  (on-site, real), and  $\chi_{i,j}$  (link, complex)

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$$Z[\varphi, \chi] = \exp(-\int_0^\beta d\tau \sum_{\langle i,j \rangle} [\frac{1}{J} |\chi_{i,j}|^2 + \ln \det \mathcal{G}_{i,j}(\tau)])$$

Saddle-point → and beyond...

# Ground state spin liquid states — Technical details

## Partition function and free energy

$$Z = \int D[c, \bar{c}] e^{-S[c, \bar{c}]}$$

$$\begin{aligned} S[c, \bar{c}] = & \int_0^\beta d\tau \left[ \sum_i \bar{c}_{i,\alpha} (\partial_\tau + \varphi_i) c_{i,\alpha} \right. \\ & \left. - J \sum_{\langle i,j \rangle} \bar{c}_{i,\alpha} c_{j,\alpha} \bar{c}_{j,\beta} c_{i,\beta} + \sum_i \varphi_i (\bar{c}_{i,\alpha} c_{i,\alpha} - 1) \right] \end{aligned}$$

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## Partition function after a Hubbard-Stratonovich transformation

$$Z = \int D[c, \bar{c}, \chi, \chi^*] e^{-S_{HS}[c, \bar{c}, \chi, \chi^*]}$$

$$\begin{aligned} S_{HS}[c, \bar{c}] = \int_0^\beta d\tau & \left[ \sum_i \bar{c}_{i,\alpha} (\partial_\tau + \varphi_i) c_{i,\alpha} \right. \\ & - \sum_{\langle i,j \rangle} \left( \chi_{i,j} \bar{c}_{j,\alpha} c_{i,\alpha} + \chi_{i,j}^* \bar{c}_{i,\alpha} c_{j,\alpha} - \frac{1}{J} |\chi_{i,j}|^2 \right) \end{aligned}$$

Integrating out the fermion fields:

$$Z = \int D[\chi, \chi^*] e^{-\int_0^\beta d\tau \sum_{\langle i,j \rangle} [\frac{1}{J} |\chi_{i,j}|^2 + \ln \det \mathcal{G}_{i,j}(\tau)]}$$

finite temperature Green's function

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Saddle-point approximation:

$$\begin{aligned}\chi_{i,j}(\hat{q}) &= \beta V \bar{\chi}_{i,j} \delta_{\hat{q},0} + \delta \chi_{i,j}(\hat{q}), \\ \varphi_i(\hat{q}) &= \beta V \bar{\varphi}_i \delta_{\hat{q},0} + \delta \varphi_i(\hat{q}).\end{aligned}$$

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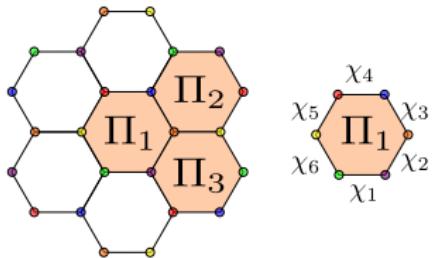
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$$\text{tr} \log(\beta \mathcal{G}^{-1}) = \text{tr} \log \left[ \beta (\mathcal{G}_{(0)}^{-1} - \Sigma) \right] = \text{tr} \log \left( \beta \mathcal{G}_{(0)}^{-1} \right) + \sum_{n=1}^{\infty} \frac{\text{tr}(\mathcal{G}_{(0)} \Sigma)^n}{n}.$$

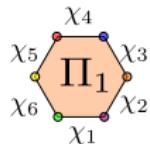
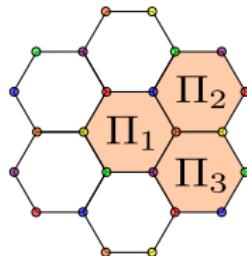
# Ground state spin liquid states



$$\chi_{i,j} = J \text{tr} \left( \mathcal{G}_0 \frac{\partial \Sigma}{\partial \chi_{i,j}^*} \right)$$

$$\Pi_1 = \chi_1 \chi_2 \chi_3 \chi_4 \chi_5 \chi_6 = |\Pi_1| e^{i\phi_1}$$

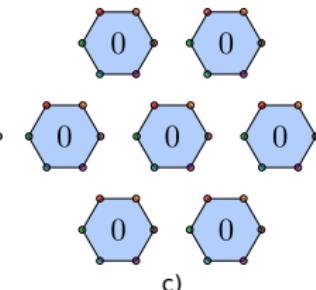
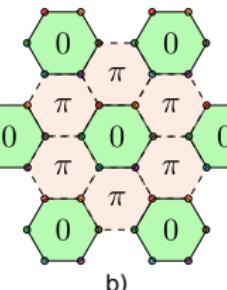
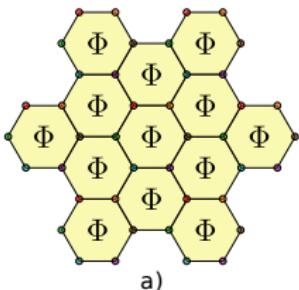
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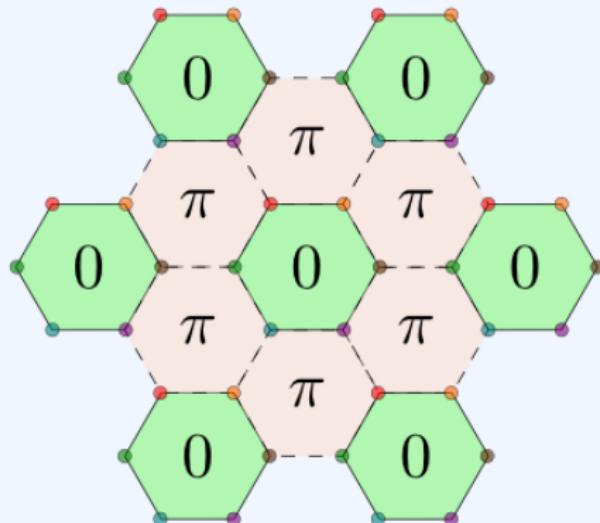
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$$\Pi_1 = \chi_1 \chi_2 \chi_3 \chi_4 \chi_5 \chi_6 = |\Pi_1| e^{i\phi_1}$$

	$E$
a) Chiral spin liquid state	-6.148
b) Straggered flux state	-6.062
c) Valence bond crystal	-6

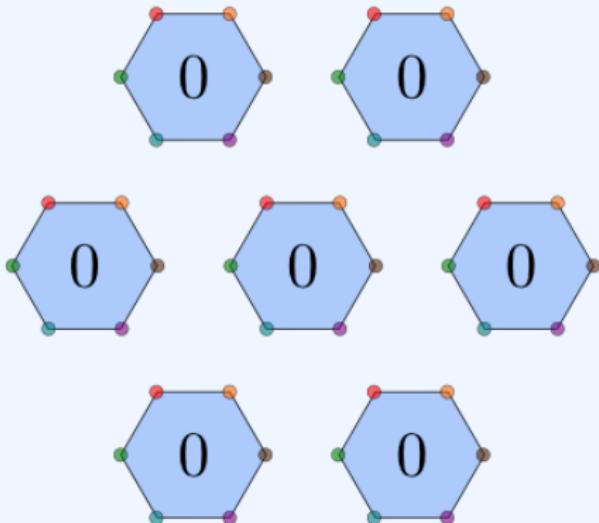


# Staggered flux state



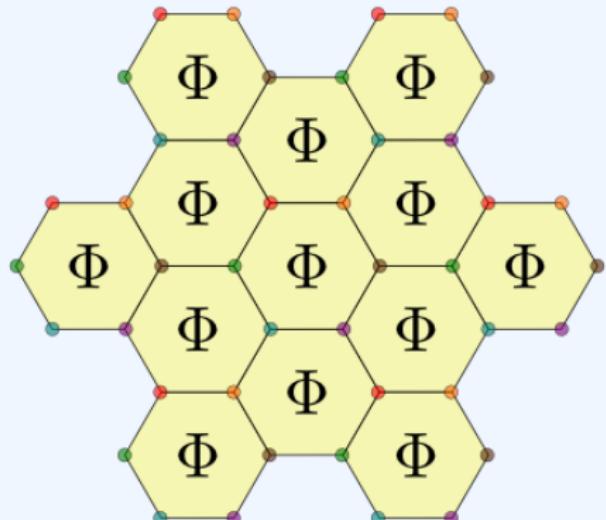
- has a triple degeneracy,
- is the honeycomb analog of the pi-flux phase
- due to the frustrated nature of the dual lattice alternating fluxes are unfavorable here.

## Box state



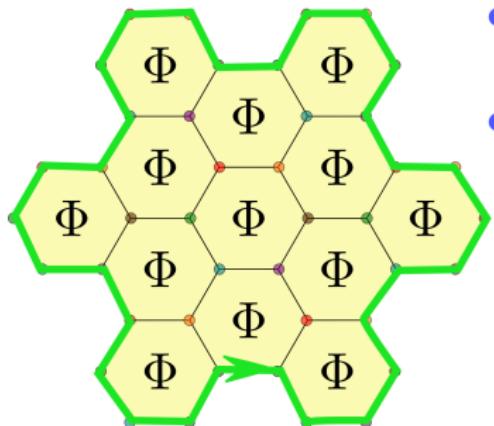
- also has a triple degeneracy,
- has zero fluxes for every plaquette,
- is composed of disjoint plaquettes,
- is the honeycomb analog of the box phase

# Chiral spin liquid state

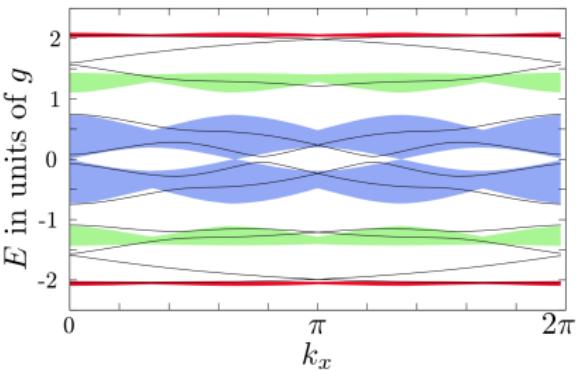


- uniform, lattice and SU(6) rotational symmetric,
- has a mean-field generated flux:  
$$\Phi = \frac{2\pi}{3},$$
- violates time reversal symmetry.

# Chiral spin liquid state



- chiral edge states appear,
- there is a nonzero transverse conductivity:  $C = 6$ ,
- quasiparticle statistics is fractionalized, each spinon carries a  $\Phi_0 = \pi/3$  elementary flux.



# Chiral spin liquid state

- The low energy fluctuations are phase fluctuations of the mean field,  $\chi_{i,j} = \chi_{i,j}^{\text{mf}} e^{ia_{i,j}}$ , and the lowest energy spinon excitations.
- $a_{i,j}$  is a gauge field, with the transformation property:

$$a_{i,j} \rightarrow a_{i,j} + \theta_i - \theta_j.$$

- The effective theory is then a U(1) Chern-Simons theory coupled to 6 spinon fields

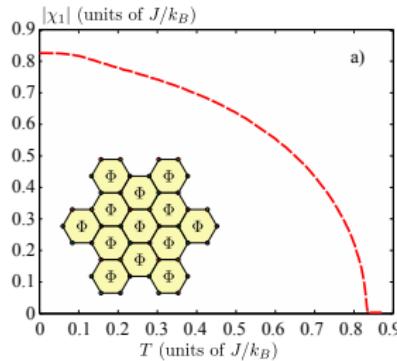
$$\begin{aligned}\mathcal{L} = & \frac{1}{8\pi q^2} (\mathbf{e}^2 - vb^2) - \frac{C}{4\pi} \epsilon^{\mu\nu\lambda} a_\mu \partial_\nu a_\lambda \\ & + \sum_{l=1}^6 \left[ -ic_{l,\alpha}^\dagger (\partial_t - ia_0) c_{l,\alpha} + \frac{1}{2m_s} c_{l,\alpha}^\dagger (\partial_i + ia_i)^2 c_{l,\alpha} \right].\end{aligned}$$

- Detection through the magnetic structure factor:

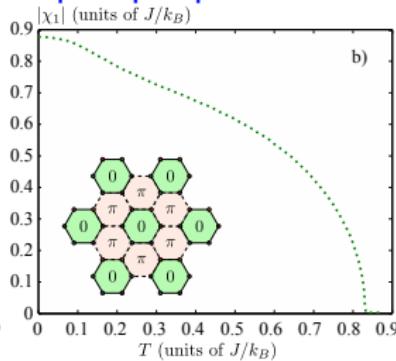
$$S^{zz}(i, j; t) = \langle S_i^z(t) S_j^z(0) \rangle$$

# Finite temperature behavior

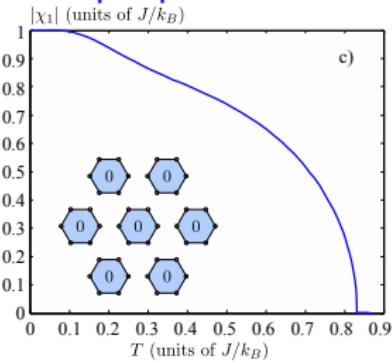
chiral state



quasiplaquette state

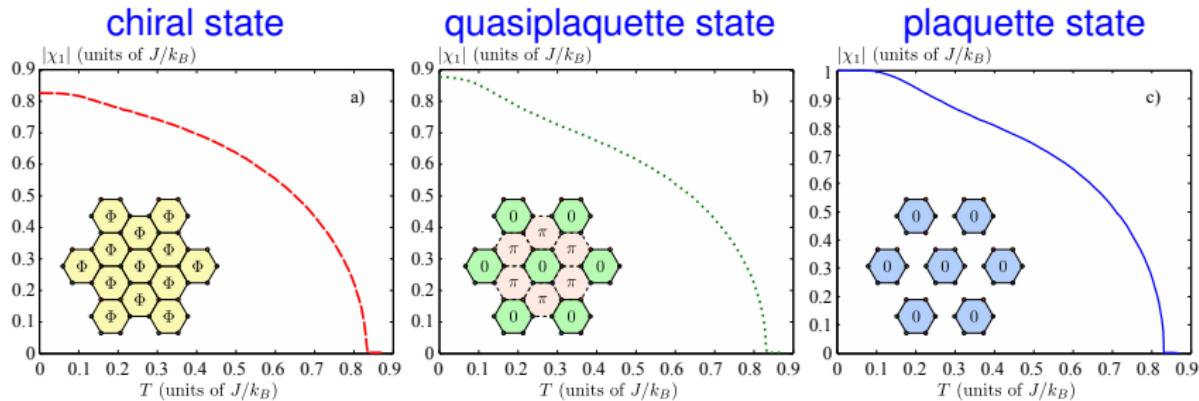


plaquette state



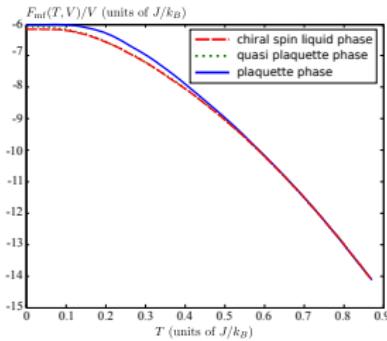
All the spin liquid phases "melt" around the same critical temperature.

# Finite temperature behavior



All the spin liquid phases "melt" around the same critical temperature.

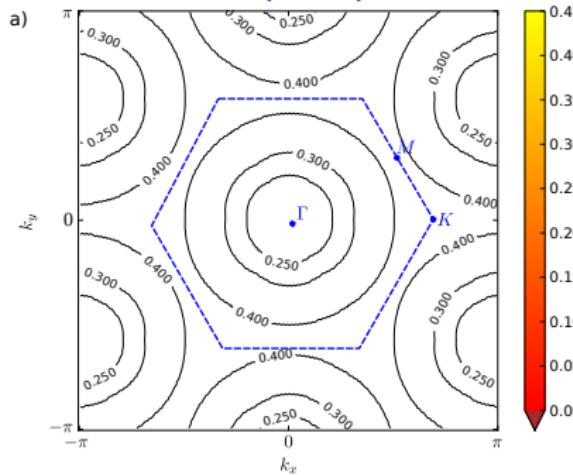
- No new state occurs as lowest free energy SP solution.
- The SP free energies approach each other without crossing.
- The chiral state remains the lowest free energy solution even at  $T > 0$ .



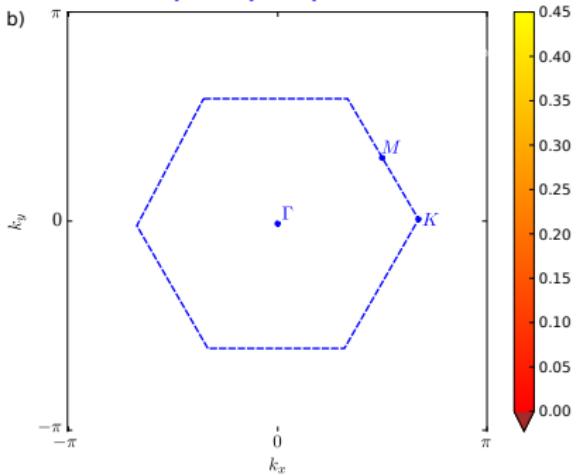
# Stability analysis

Stability matrix :  $C_{\mu,\nu} \sim \frac{\partial^2 (\mathcal{G}_0 \Sigma)^2}{\partial \chi_\mu \partial \chi_\nu}$

chiral spin liquid



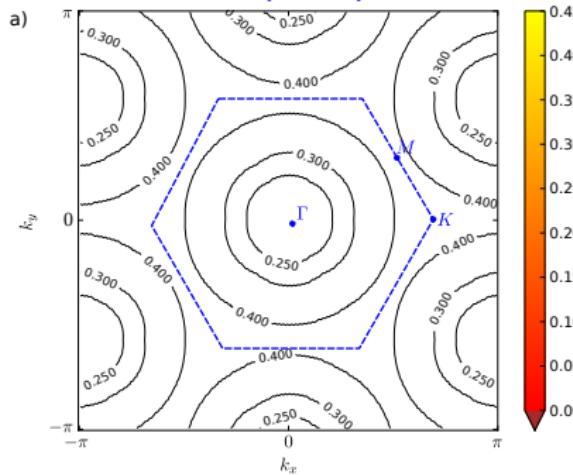
quasiplaquette



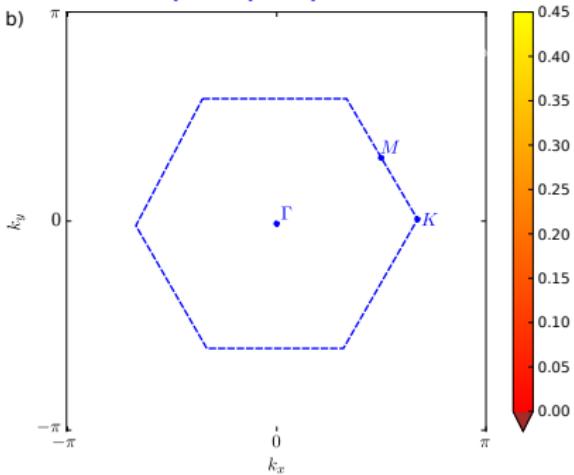
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chiral spin liquid



quasiplaquette

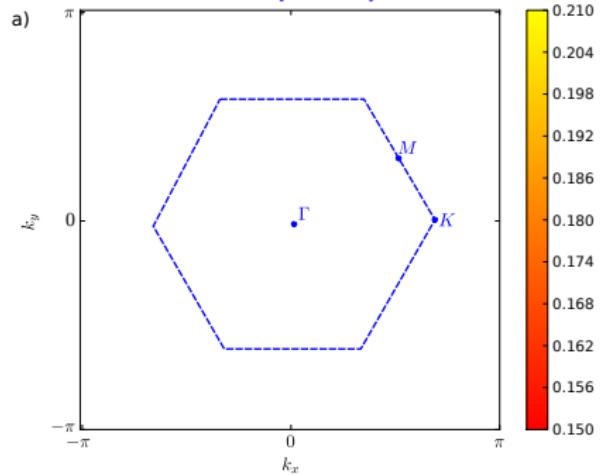


The quasiplaquette state collapses into the lower free energy chiral spin liquid state.

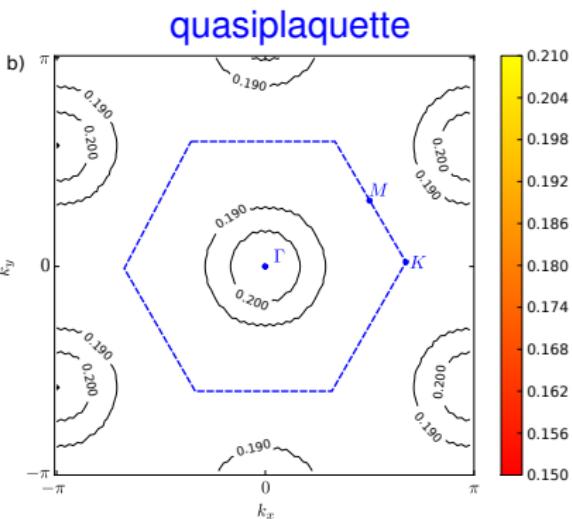
# Experimentally measurable quantities

$$\text{Structure factor: } S(\mathbf{r}, \tau; \mathbf{r}', 0) = \langle S_z(\mathbf{r}, \tau) S_z(\mathbf{r}', 0) \rangle$$

chiral spin liquid



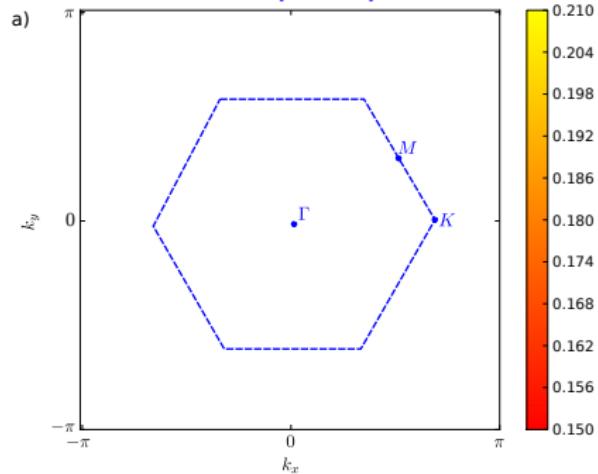
quasiplaquette



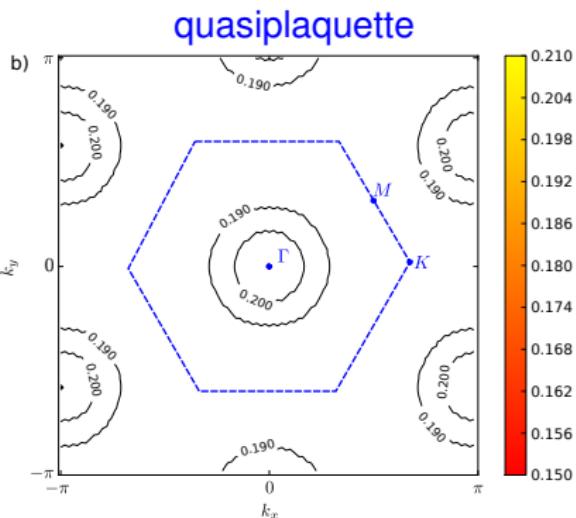
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chiral spin liquid



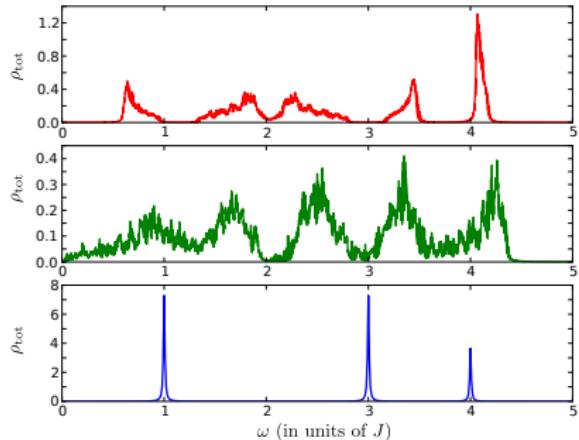
quasiplaquette



Unambiguous features → Suitable tool to distinguish the phases.

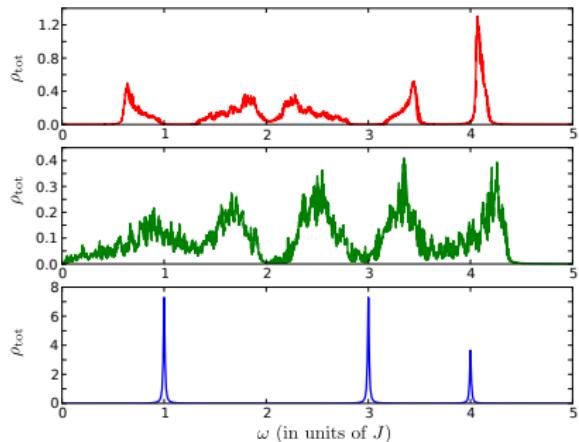
# Experimental measurable quantities

Spectral density:  $\rho_{\text{tot}}(\omega) = \sum_{\mathbf{k}} \text{Im}S(\mathbf{k}, \omega)$



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Thank you for your attention