Stability of spin liquid phases of high spin alkaline earth atoms on honeycomb lattice

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Workshop on Coherent Control of Complex Quantum Systems
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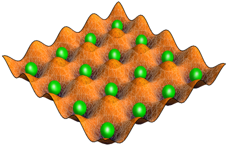
In collaboration with:
M. Lewenstein, P. Sinkovicz, G. Szirmai, A. Zamora

Quantum simulations with ultracold atoms



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- Novel behavior, completely new phases are expected due to the high spin.

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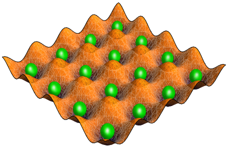

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In topological phases of spin liquids the quasiparticles have fractional statistics. They are nonlocal and resist well against local perturbations. Promising qbit candidates.

- Quantum information

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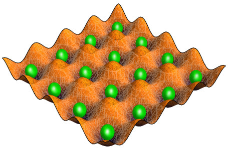

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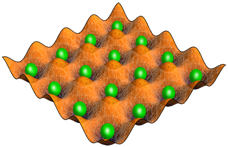
Low energy excitations above spin liquids can be described by effective gauge theories. Aim: to study various gauge theories with ultracold atoms.

- Quantum information
- Simulation of gauge theories

Quantum simulations with ultracold atoms



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A possible explanation for the mechanism of high-T_c superconductivity and their strange behavior in the non-superconducting phase based on the strong magnetic fluctuation in dopped Mott insulators. These fluctuation can be treated within the spin liquid concept.

- Quantum information
- Simulation of gauge theories
- High-T_c superconductors

Quantum simulations with ultracold atoms

Spin-5/2 fermions

- Spin-5/2 ultracold atom experiments with ^{173}Yb.
- Various manganise complexes; $NaReO_4$, NH_4ReO_4 with ^{185}Re or ^{187}Re etc.

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- Quantum information
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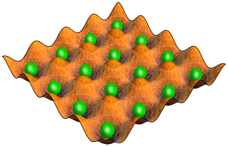
- High-T_c superconductors

Outline

- Simulation of high spin magnetism
- Competing spin liquid states of spin-5/2 fermions in a honeycomb lattice
    - Properties of chiral spin liquid state
    - Stability of the spin liquid states beyond the mean-field approximation
    - Finite temperature behavior
    - Experimentally measurable quantities
- Summary



Simulation of high spin magnetism

Atoms loaded into an optical lattice

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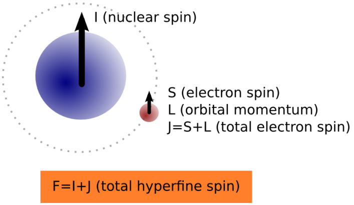

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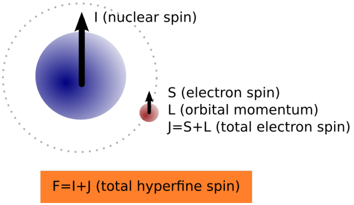

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Easy to control the model parameters:

- Interaction strength: Feshbach resonance
- Localization: laser intensity
- Lattice geometry

Simulation of high spin magnetism

The Hubbard Hamiltonian with n.n hopping and on-site interaction:

$$H = -t \sum_{<i,j>} c^{\dagger}_{i,\alpha} c_{j,\alpha} + U \sum_{i} c^{\dagger}_{i,\alpha} c^{\dagger}_{i,\beta} c_{i,\alpha} c_{i,\beta}.$$

spin independent interaction $\rightarrow$ SU(N) symmetry

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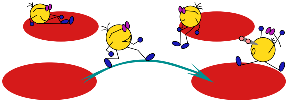
Strongly repulsive limit: $U/t \rightarrow \infty$

repulsive interaction, $f = 1/(2F+1)$ filling ($\#$ of particles $= \#$ of sites)

Perturbation theory up to leading order with respect to t.

- t preserves S and m_S
- nearest-neighbor hopping



Simulation of high spin magnetism

Effective Hamiltonian (spin-F fermions in the $U/t \to \infty$ limit)

$$H = J \sum_{\langle i,j \rangle} c^{\dagger}_{i,\alpha} c^{\dagger}_{j,\beta} c_{j,\alpha} c_{i,\beta}$$

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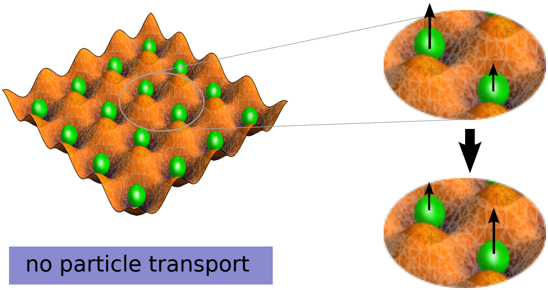

no particle transport

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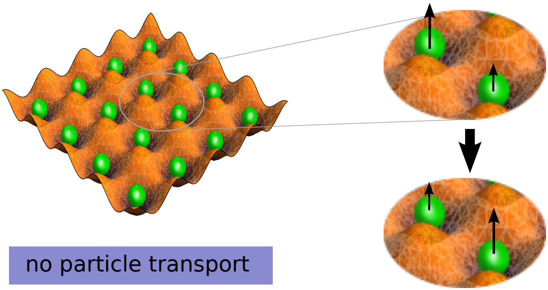

no particle transport

Without long range magnetic order: spin liquid state

Competing spin liquid states of spin-5/2 fermions on honeycomb lattice

Finite temperature field theory

$$H[c, c^\dagger] = -J \sum_{\langle i,j \rangle} c^\dagger_{i,\alpha} c_{j,\alpha} c^\dagger_{j,\beta} c_{i,\beta}$$

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$L[c,\bar{c}] = \sum_i \bar{c}_{i,\alpha} \partial_\tau c_{i,\alpha} + H$ and #atoms= #sites

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Decoupling procedure:

Hubbard-Stratonovich transformation: $L[c,\bar{c}] \to L[c,\bar{c};\varphi,\chi]$

auxiliary fields: φ_i (on-site, real), and $\chi_{i,j}$ (link, complex)

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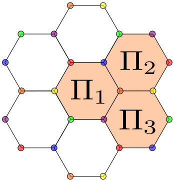
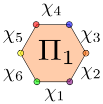
$$Z[\varphi,\chi] = \exp(-\int_0^\beta \mathrm{d}\tau \sum_{\langle i,j\rangle} \left[\tfrac{1}{J}|\chi_{i,j}|^2 + \ln\det\mathscr{G}_{i,j}(\tau)\right])$$

Saddle-point approximation $\to$ and beyond...

Ground state spin liquid states





$$\chi_{i,j} = J \,\mathrm{tr}\left(\mathscr{G}_0 \frac{\partial \Sigma}{\partial \chi^*_{i,j}} \right)$$

(Σ: self-energy)

$$\Pi_1 = \chi_1 \chi_2 \chi_3 \chi_4 \chi_5 \chi_6 = |\Pi_1| \mathrm{e}^{\mathrm{i}\phi_1}$$

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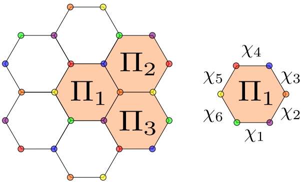

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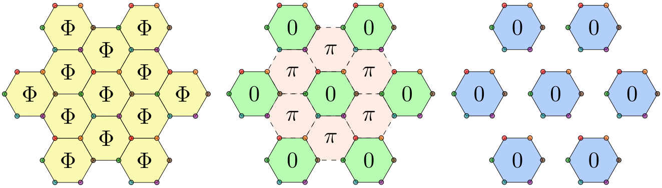
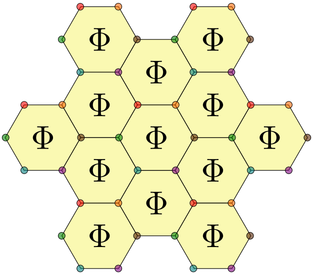
	E
a) Chiral spin liquid state	-6.148
b) Straggered flux state	-6.062
c) Valence bond crystal	-6



a) b) c)

Ground state — chiral spin liquid

- Preserves the global SU(6) invariance of the Hamiltonian and every lattice symmetry.
- $\Phi = 2\pi/3$ flux generated per plaquette $\Rightarrow$ spontenous time reversal symmetry breaking.



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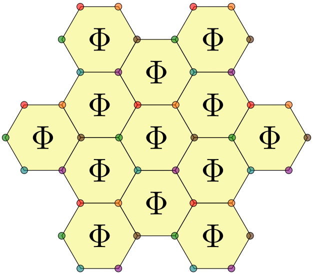
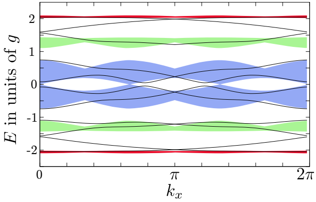


Consequences:

- Integer quantum-Hall effect, transverse conductivity $C = 6$.
- Chiral edge states appear.
- Anyon quasiparticles: spinon with $\Phi_0 = \pi/3$ elementary flux.



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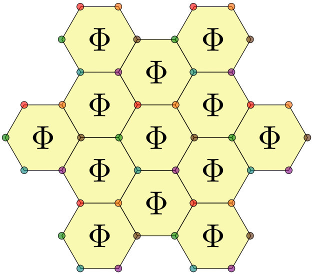

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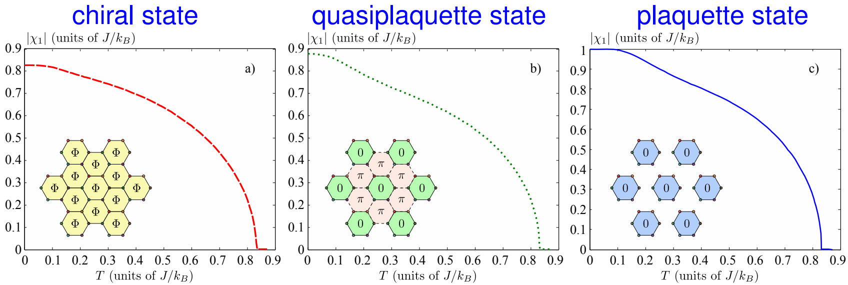
And

The low energy effective theory is a U(1) gauge theory: Chern-Simons theory.

U(1) gauge theory simulator.

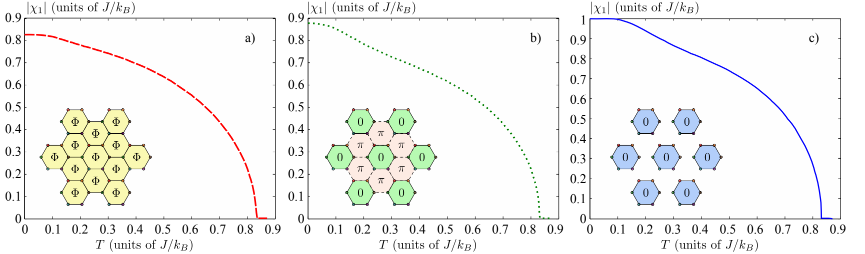
Finite temperature behavior



All the spin liquid phases "melt" around the same critical temperature.

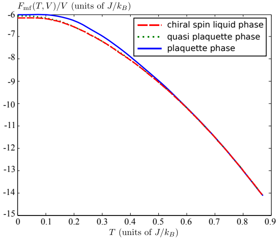
Finite temperature behavior

chiral state | quasiplaquette state | plaquette state



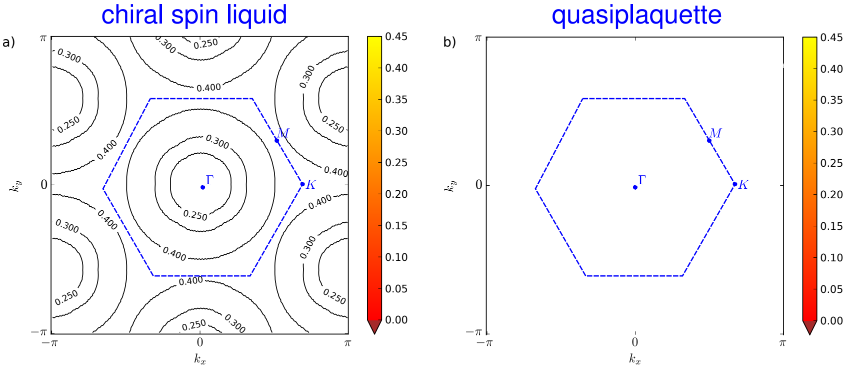
All the spin liquid phases "melt" around the same critical temperature.

- No new state occurs as lowest free energy SP solution.
- The SP free energies approach each other without crossing.
- The chiral state remains the lowest free energy solution even at $T > 0$.



Stability analysis

$$\text{Stability matrix}: \quad C_{\mu,\nu} \sim \frac{\partial^2 (\mathscr{G}_0 \Sigma)^2}{\partial \chi_\mu \partial \chi_\nu}$$



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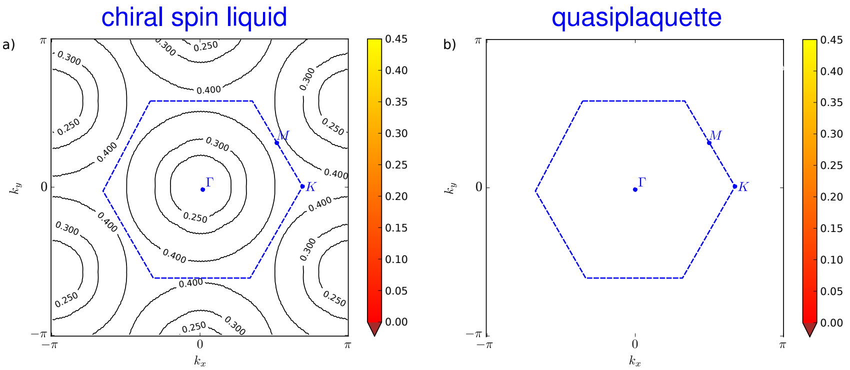
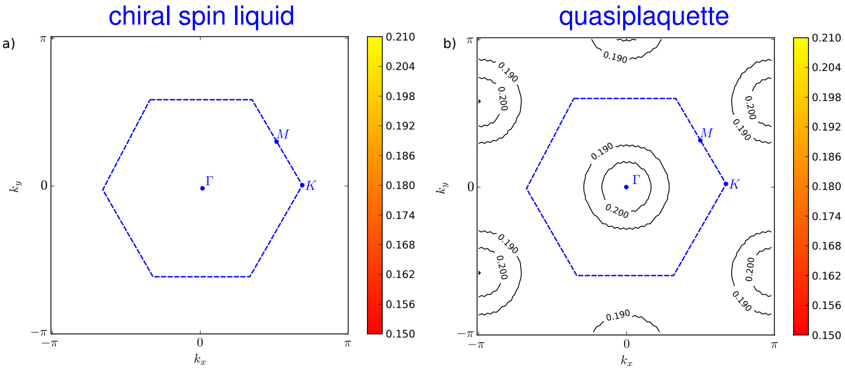


The quasiplaquette state collapses into the lower free energy chiral spin liquid state.

Experimentally measurable quantities

Structure factor: $S(\mathbf{r},\tau;\mathbf{r}',0) = \langle S_z(\mathbf{r},\tau) S_z(\mathbf{r}',0) \rangle$



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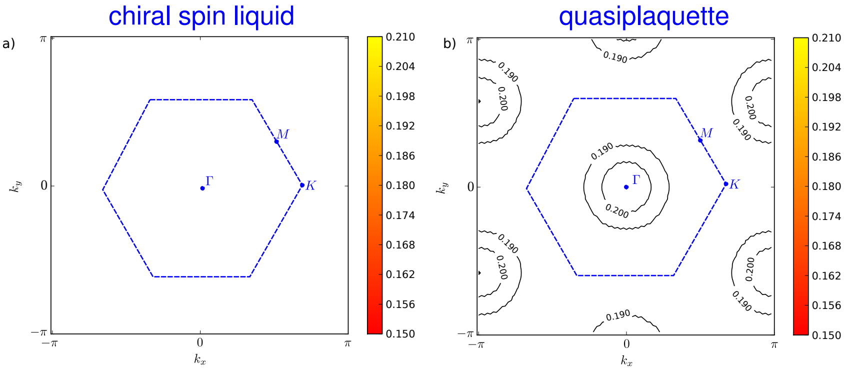
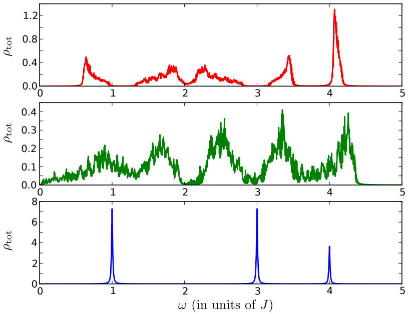


Unambiguous features $\longrightarrow$ Suitable tool to distinguish the phases.

Experimental measurable quantities

Spectral density: $\rho_{\text{tot}}(\omega) = \sum_{\mathbf{k}} \mathrm{Im}\, S(\mathbf{k}, \omega)$



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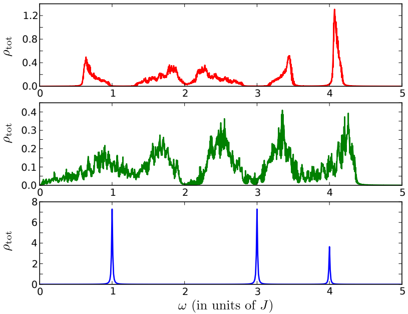

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Summary

- Time-reversal symmetry breaking spin liquid state of spin-5/2 fermionic atoms on honeycomb lattice.
- Stable even at finite temperature.
- Experimentally probable.
- $\rightarrow$ Simulation of a U(1) gauge theory.

G. Szirmai, E. Sz., A. Zamora, M. Lewenstein, PRA **84** 011611(R) (2011)

P. Sinkovicz, A. Zamora, E. Sz. M. Lewenstein, G. Szirmai, PRA **88** 043619 (2013)