Exotic magnetic orders for high spin ultracold fermions

Workshop on Correlations and Coherence in Quantum Systems

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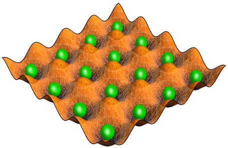
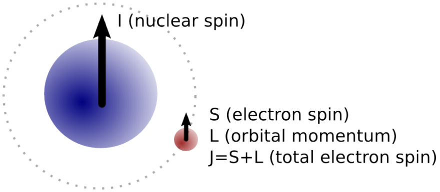
Edina Szirmai (BUTE, Budapest, Hungary)
Maciej Lewenstein (ICFO, Castelldefels, Spain)

Outline



- High-spin (ultracold) atoms
- Strongly repulsive limit
- Effective models
- Mean-field phase diagram of spin-3/2 fermions on square lattice
- Summary

Ultracold atoms as simulator of fundamental models



F=I+J (total hyperfine spin)

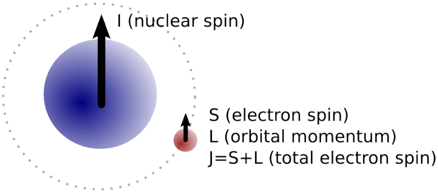
- Quantum simulation of fundamental models (properties, phenomena).
- Novel behavior, completely new phases are expected due to the high spin.

spin-3/2 fermions

- UCA experiments : ^{6}Li, ^{132}Cs, ^{9}Be, ^{135}Ba, ^{137}Ba;
- $Ba_2CoGe_2O_7$ (square lattice), $Bi_3Mn_4O_{12}(NO_3)$ (double layered hexagonal);

If we have a non-SU(N) symmetric high spin system the life is much more complicated.

Ultracold atoms as simulator of fundamental models



F=I+J (total hyperfine spin)

- Quantum simulation of fundamental models (properties, phenomena).
- Novel behavior, completely new phases are expected due to the high spin.

5/2 spinű fermionok

- UCA experiments: ^{173}Yb, ^{52}Cr;
- different Mn-compounds ; $NaReO_4$, NH_4ReO_4 (with ^{185}Re or ^{187}Re);

If we have a non-SU(N) symmetric high spin system the life is much more complicated.

Ultracold atoms as simulator of fundamental models

The Hamiltonian:

$$H = H_{kin} + H_{int}$$

where H_{int} contains many types of scattering processes.

on-site interaction

$\Longrightarrow$

Pauli's principle

The only nonzero terms are complety antisymmetric for the exchange of the spin of the two scattering particles.

spin-3/2 fermions

Classification of the allowed scattering processes:

- $S_{tot} = 0$, singlet
- $S_{tot} = 2$, quintet

$\Longrightarrow$

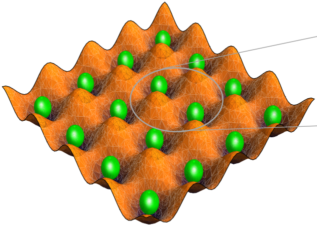
$H = -t\sum_{\langle i,j\rangle}(c^\dagger_{i,\sigma}c_{j,\sigma} + H.c.) + \sum_i \left[U_0 \mathscr{P}^{(i)}_0 + U_2 \mathscr{P}^{(i)}_2\right]$,

and $\mathscr{P}^{(i)}_S = c^\dagger_{i,\sigma_1} c^\dagger_{i,\sigma_2} c_{i,\sigma_3} c_{i,\sigma_4} \hat{P}_S$, where $\hat{P}_S$ projects to the subspace $S_{tot} = S$.

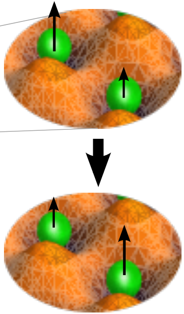
T. L. Ho PRL (1998)

T. Ohmi and K. Machida JPSJ (1998)

Strong coupling limit: $U_S/t \to \infty$



no particle transport

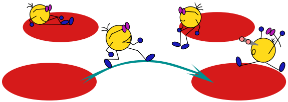


If there is no magnetic order and the lattice symmetries are preserved: spin liquid state.

Strong coupling limit: $U_S/t \to \infty$

Perturbation theory up to leading order with respect to t.

- t preserves S and m_S
- nearest-neighbor hopping



Effective Hamiltonian

$$H = \sum_{\langle i,j \rangle} \left[g_0 \mathscr{P}_0^{(i,j)} + g_2 \mathscr{P}_2^{(i,j)} \right]$$

- nearest-neighbor interaction
- the same spin dependence that has the original model

where $\mathscr{P}_S^{(i,j)} = c^\dagger_{i,\sigma_1} c^\dagger_{j,\sigma_2} c_{j,\sigma_3} c_{i,\sigma_4} \hat{P}_S$ and $g_S = -4t^2/U_S$.

$$H_{eff} = \sum_{\langle i,j \rangle} \left[a_0\, n_i n_j + a_1 \mathbf{S}_i \mathbf{S}_j + a_2 (\mathbf{S}_i \mathbf{S}_j)^2 + a_3 (\mathbf{S}_i \mathbf{S}_j)^3 \right]$$

$$n_i = c^\dagger_{i,\sigma} c_{i,\sigma}, \text{ and } \mathbf{S}_i = c^\dagger_{i,\sigma} \mathbf{F}_{\sigma,\sigma'} c_{i,\sigma'}.$$

The general form of the projectors:

$$\hat{P}_S = \sum_{l=0}^{2F} a_{S,l}(\mathbf{F}_1\mathbf{F}_2)^l \text{ with } (\mathbf{F}_1\mathbf{F}_2)^0 = \mathbf{E}_{1,2}$$

BUT

$\hat{P}_S$ is antisymmetric in the spin indices of the scattering particles!

$$(\mathbf{F}_1\mathbf{F}_2)^l = \left((\mathbf{F}_1\mathbf{F}_2)^l\right)^{(s)} + \left((\mathbf{F}_1\mathbf{F}_2)^l\right)^{(as)}$$

$$\hat{P}_S = \sum_{l=0}^{2F} b_{S,l}\left((\mathbf{F}_1\mathbf{F}_2)^l\right)^{(as)}$$

spin-3/2 fermions

$$H_{int} = \sum_{<i,j>}\left[a_n\mathbf{E}_{i,j}^{(as)} + a_s(\mathbf{F}_1\mathbf{F}_2)_{i,j}^{(as)}\right]$$ with $a_n = (5G_2 - G_0)/4$, $a_s = (G_2 - G_0)/3$, and

$$\mathbf{E}_{i,j}^{(as)} = c_{i,\alpha}^\dagger c_{j,\beta}^\dagger c_{j,\delta} c_{i,\gamma}\left[\mathbf{E}^{(as)}\right]_{\gamma,\delta}^{\alpha,\beta}, \text{ and } (\mathbf{F}_1\mathbf{F}_2)^{(as)} = c_{i,\alpha}^\dagger c_{j,\beta}^\dagger c_{j,\delta} c_{i,\gamma}\left[(\mathbf{F}_1\mathbf{F}_2)^{(as)}\right]_{\gamma,\delta}^{\alpha,\beta},$$

$$\left[\mathbf{E}^{(as)}\right]_{\gamma,\delta}^{\alpha,\beta} = \delta_{\alpha,\gamma}\delta_{\beta,\delta} - \delta_{\alpha,\delta}\delta_{\beta,\gamma}, \text{ and } \left[(\mathbf{F}_1\mathbf{F}_2)^{(as)}\right]_{\gamma,\delta}^{\alpha,\beta} = [\mathbf{F}_1]_{\alpha,\gamma}[\mathbf{F}_2]_{\beta,\delta} - [\mathbf{F}_1]_{\alpha,\delta}[\mathbf{F}_2]_{\beta,\gamma}.$$

The new effective Hamiltonian:

$$H_{eff} = \sum_{<i,j>}\left[a_n\left(n_i n_j + \chi^\dagger_{i,j}\chi_{i,j} - n_i\right) + a_s\left(\mathbf{S}_i\mathbf{S}_j + \mathbf{B}^\dagger_{i,j}\mathbf{B}_{i,j} - \frac{15}{4}n_i\right)\right]$$

E. Sz. and M. Lewenstein EPL **93** 66005 (2011)

Site- and bond-centered orders

- $n_i = c^\dagger_{i,\alpha}c_{i,\alpha}$ (particle number at site i)
- $\mathbf{S}_i = c^\dagger_{i,\alpha}\mathbf{F}_{\alpha,\beta}c_{i,\beta}$ (spin at site i)
- $\chi_{i,j} = c^\dagger_{i,\alpha}c_{j,\alpha}$ (BCDW)
- $\mathbf{B}_{i,j} = c^\dagger_{i,\alpha}\mathbf{F}_{\alpha,\beta}c_{j,\beta}$ (BSDW)

- $|\chi_{i,j}|^2 \propto \langle\mathbf{S}_i\mathbf{S}_j\rangle \;\to\;$ spin-Peierls distortion B. Marston and J. Affleck PRB (1989)
- $|\mathbf{B}_{i,j}|^2 \propto \langle\mathbf{Q}_i\mathbf{Q}_j\rangle \;\to\;$ quadrupole-Peierls distortion

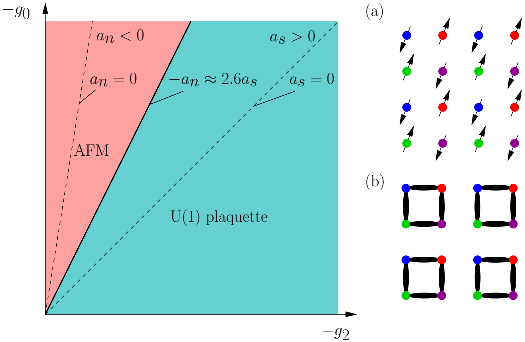
Mean-field phase diagram: $F = 3/2$, 1/4 filling, square lattice

Parameters

$$a_n = \frac{5g_2 - g_0}{4}$$

$$a_s = \frac{(g_2 - g_0)}{3}$$

$$g_S = -\frac{4t}{U_S}$$



Nonzero order parameters

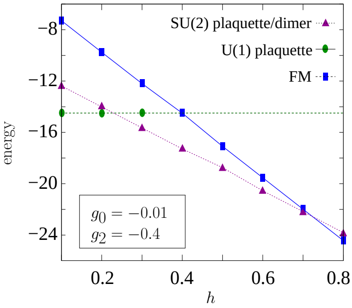
- AFM: $\langle \mathbf{S}_i \rangle$
- U(1) plaquette: $\langle \chi_{i,j} \rangle$ ($\Phi = 0$, or $\pm\pi$; VBS)

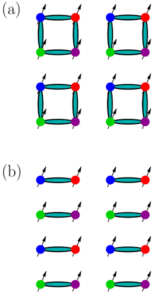
For the SU(4) line: C. Wu MPL (2006)
E. Sz. and M. Lewenstein EPL **93** 66005 (2011)

In presence of magnetic field

Hamiltonian with magnetic field:

$$H^h = H^{MF} + h\sum_i \mathbf{S}_i.$$





Order parameters

- SU(2) plaquette: $\langle \mathbf{S}_i \rangle$, $\langle \chi_{i,j} \rangle$, $\langle \mathbf{B}_{i,j} \rangle$
- SU(2) dimer: $\langle \mathbf{S}_i \rangle$, $\langle \chi_{i,j} \rangle$, $\langle \mathbf{B}_{i,j} \rangle$

E. Sz. and M. Lewenstein EPL **93** 66005 (2011)

Summary

- We considered high-spin fermion systems on lattice.
- With a simple treatment a novel effective model was introduced that able to describe the competition between the site and bond order (even on mean-field level).
- This description can be interpreted within the concept of site and bond multipoles.
- We determined the mean-field phase diagram of the spin-3/2 fermion system on square lattice at 1/4 filling and we showed that the exotic SU(2) plaquette state can have the lowest energy in presence of external magnetic field. This state can be characterized by quadrupole-Peierls distorsion.